# EEE225: Analogue and Digital Electronics

## Lecture II

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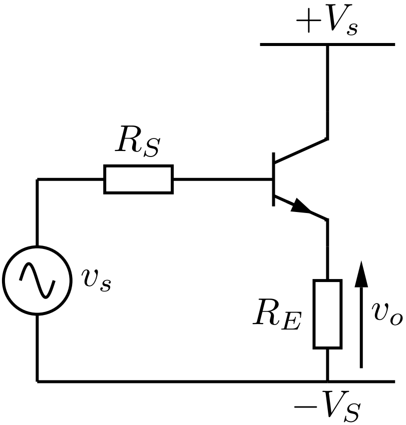
# This Lecture

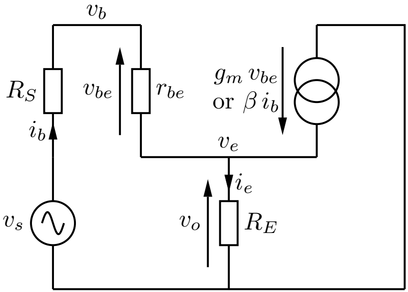
1. One Transistor Circuits Continued...
    - Emitter Follower or Common Collector
    - Emitter Follower Voltage Gain
    - Emitter Follower Input Resistance
    - Emitter Follower Output Resistance
    - Common Base
    - Common Base Voltage Gain
    - Common Base Input Resistance
2. Inside the Opamp
    - Feedback System
    - Simplified Schematic of an Opamp
    - Opamp Circuit DC Conditions
    - Differential Amplifier
3. Review
4. Bear

# Emitter Follower / Common Collector

- A kind of "voltage follower" or "buffer"
- Approximately unity voltage gain
- pnp or npn versions possible
- High current gain
- May be thought of as impedance transformer (so can all transistor circuits...)



In this figure the biasing circuitry is contained as an effective resistance within $R_S$



$$v_o = v_{be}\,R_E\left(\frac{1}{r_{be}} + g_m\right) \tag{1}$$

$$\approx v_{be}\,R_E\,g_m \tag{2}$$

and a relation between $v_{be}$, $v_s$ and $v_o$ is given by summing voltages around the input loop.

$$v_s = i_b\,R_S + v_{be} + v_o \tag{3}$$

$$= v_{be}\left(1 + \frac{R_S}{r_b e}\right) + v_o \tag{4}$$

using the result in (2) to eliminate $v_{be}$,

$$\frac{v_o}{v_s} = \frac{r_{be}\,g_m\,R_E}{r_{be}\,g_m\,R_E + R_S + r_{be}} \tag{5}$$

$$= \frac{R_E}{\frac{1}{g_m} + \frac{R_S}{\beta} + R_E} \tag{6}$$

1. The gain is non-inverting
2. Gain $\approx 1$ if $R_E >> R_S/\beta$ and $R_E >> 1/g_m$

The input resistance is given by considering $v_b/i_b$, recall (1)

$$v_e = v_{be}\, R_E \left(\frac{1}{r_{be}} + g_m\right) \qquad (7)$$

and summing up the voltages...

$$v_b = v_{be} + v_e \qquad (8)$$

$$= v_{be} + v_{be}\, R_E \left(\frac{1}{r_{be}} + g_m\right) \qquad (9)$$

$$= v_{be} \left(1 + R_E \left(\frac{1}{r_{be}} + g_m\right)\right) \qquad (10)$$

since $v_{be} = i_b\, r_{be}$ and $g_m\, r_{be} = \beta$ we can write,

$$r_i = \frac{v_b}{i_b} = r_{be} + (\beta + 1)\; R_E \qquad (11)$$

Generally $(\beta + 1)\, R_E >> r_{be}$ so the input resistance is dominated by the $(\beta + 1)\, R_E$ term. By comparing this result with the input resistance of the non-degenerated common emitter amplifier we could show negative feedback can be used to increase the input resistance of a transistor stage.

To obtain the output resistance inject a test current $i_t$ with the input grounded and find $v_o/i_t$. Summing currents at $v_e$

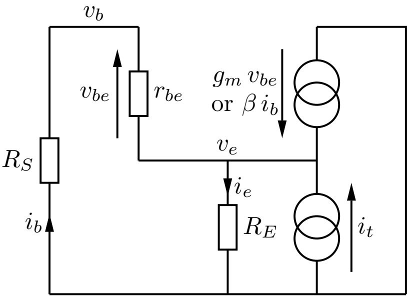
$$(1+\beta)\; i_b + i_t = \frac{v_e}{R_E} \qquad (12)$$

and summing up the voltages in the base loop

$$v_e = -i_b\,(R_S + r_{be}) \qquad (13)$$

substituting (13) into (12) and solving for $v_e/i_t$,

$$r_o = \frac{1}{\frac{1+\beta}{R_S+r_{be}}} + \frac{1}{R_E} \qquad (14)$$

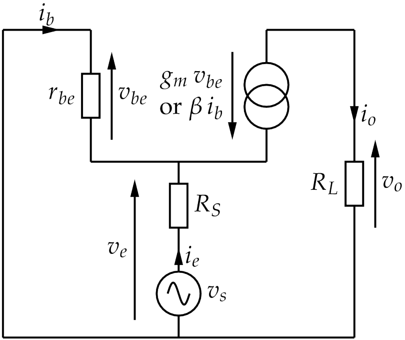


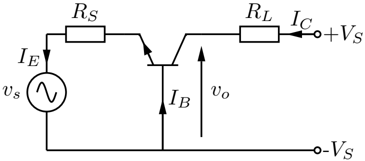
$$r_o \approx \frac{1}{g_m} + \frac{R_S}{\beta} \qquad (15)$$

If $\beta >> 1$, the first term becomes $\frac{R_S+r_{be}}{\beta}$ and if $R_E$ is large, we can ignore the $\frac{1}{R_E}$ term.

# Common Base Connection

Generally used in conjunction with other transistors in “circuit blocks”, but sometimes alone[1]. $i_e$ is the input current (flowing from $v_s$), since $i_e = i_o + i_b$ the current gain ($i_o/i_e$) is slightly less than 1 (actually it's $= \alpha$).





summing currents,

$$i_e + i_b + g_m\, v_{be} = 0 \tag{16}$$

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[1] http://dx.doi.org/10.1088/0957-0233/23/12/125901

$$\frac{v_s - v_e}{R_S} + \frac{v_{be}}{rbe} + g_m \, v_{be} = 0 \quad (17)$$

$v_e + v_{be} = 0$ so $v_e = -v_{be}$ therefore (17) can be solved for $v_{be}$

$$v_{be} = -\frac{v_s}{R_S \left( \frac{1}{R_S} + \frac{1}{r_{be}} + g_m \right)} \quad (18)$$

$$\approx -\frac{v_s}{1 + g_m \, R_S} \quad (19)$$

approximation is because $1/r_{be} = g_m/\beta$ and $\beta >> 1$

At the output,

$$v_o = i_o \, R_L = -g_m \, v_{be} \, R_L \quad (20)$$

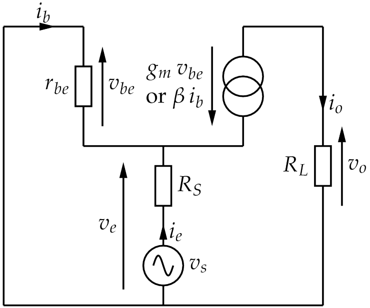
combining this with (19) to eliminate $v_{be}$

$$\frac{v_o}{v_s} = \frac{g_m \, R_L}{1 + g_m \, R_S} = \frac{R_L}{r_e + R_S} \quad (21)$$

where $r_e = 1/g_m$.

- The gain is non-inverting
- Gain $\propto R_L$
- If $R_S >> r_e$ gain controlled by ratio $R_L/R_S$

# Common Base Input Resistance



The resistance looking into the emitter,

$$r_i = \frac{v_e}{i_e} = \frac{v_e}{\frac{-v_{be}}{r_{be}} - g_m\, v_{be}} \qquad (22)$$

Since $v_e = -v_{be}$ and $g_m >> 1/r_{be}$ this reduces to $r_i \approx \frac{1}{g_m} = r_e$ The value is small 10s - 100s $\Omega$

There is another model of the transistor called "T Model" in which $r_e$ plays a much bigger role. However hybid-$\pi$ is the only model we will use. The original $\pi$ paper is by Giacolletto[2].

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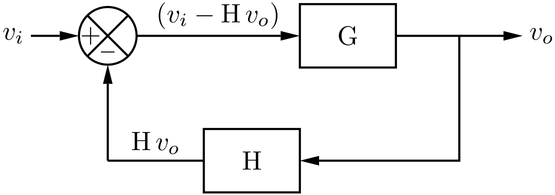
[2] http://dx.doi.org/10.1109/JSSC.1969.1049963

# Feedback Systems (Quick reminder)

In EEE118 we discussed the opamp in terms of a general feedback system.



$$\text{So } v_o = G\,(v_i - H\,v_o) \qquad (23)$$

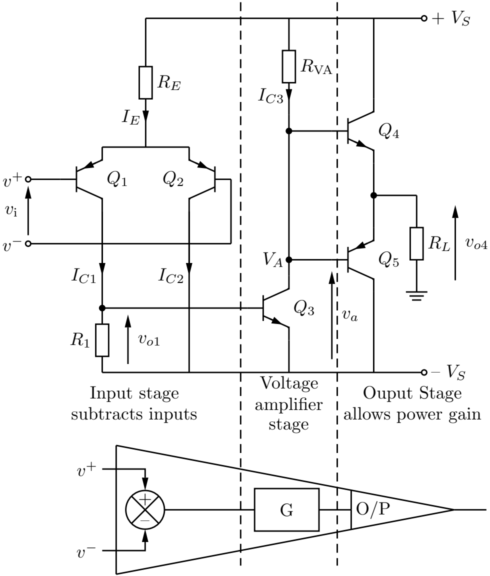
$$\text{or } v_o\,(1 + G\,H) = G\,v_i \qquad (24)$$

$$\frac{v_o}{v_i} = \frac{G}{1 + G\,H} \qquad (25)$$

If $|G\,H| >> 1$,

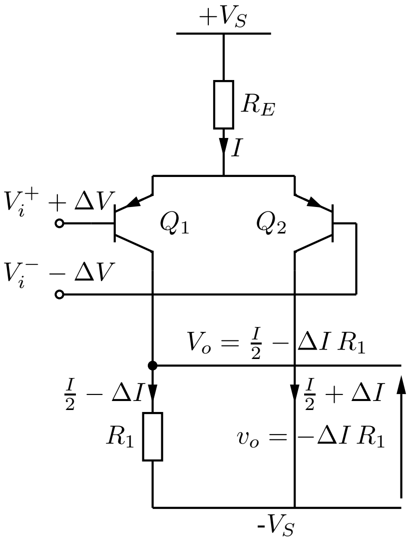
$$\frac{v_o}{v_i} = \frac{G}{G\,H} = \frac{1}{H} \qquad (26)$$

System dependent on H, designer controls $H$ with ratio of resistors.

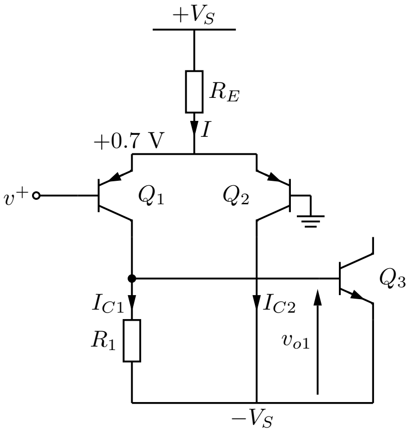


- Input stage: differential amplifier or “long tailed pair”. Subtracts the inputs.
- Voltage amplifier stage (VAS): common emitter amplifier. Provides majority of voltage gain.
- Output stage: emitter follower. Increases current capability of VAS (voltage × current = power... hence “power gain”.

- Opamp will not work properly without feedback. Feedback controls the gain of the circuit but also helps define the DC conditions. Feedback adjusts $v_i$ in order to achieve the internal voltage drops required for proper operation. If $v_o = 0$, $v_i$ will be at the value it needs to be in order to make $v_o = 0$. Feedback is *not* shown on prior slide.
- If $v^+ \approx v^- \approx 0$, $V_{E1}$ and $V_{E2} \approx 0.7$ so $I_E \approx (+V_S - 0.7)/R_E$.
- $I_E$ splits between $Q_1$ and $Q_2$ to form $I_{C1}$ and $I_{C2}$.
- $I_{C1}$ has two functions 1) create a voltage drop of 0.7 V across $R_1$ in order to bias $Q_3$ into conduction. 2) Provide the base current for $Q_3$. $I_{C1}$ will be $0.7/R_1 + I_{C3}/h_{FE3}$.
- The value of $I_{C3}$ varies with $V_A$ and hence with $V_{o4}$ but assuming $V_A = 0$, $I_{C3} = +V_S/R_{VA}$.
- $I_{C2}$ is returned directly to the negative supply.
- In the case where $v^+ \approx v^- \neq 0$, there is a common mode input voltage, $v_{cm}$, and $I_E \approx (+V_S - v_{cm})/R_E$.

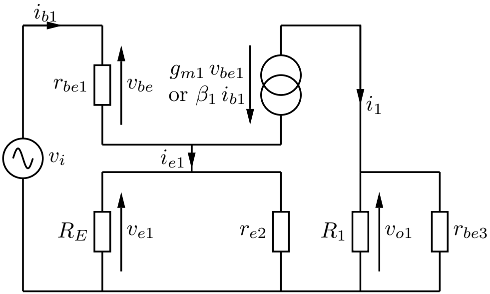


- If $v^+$ increases by $\Delta v_i$ and $v^-$ decreases by $\Delta v_i$, the average of $v^+$ and $v^-$ is unchanged so $I_E$ is unchanged because $V_{be}$ is unchanged.
- If $v^+$ and $v^-$ increase or decrease by $\Delta v_i$, $v_i$ is called a “common mode signal” ideally the differential amplifier will not amplify any common mode component of the input.



We must consider the effects of three transistors. $Q_1$ and $Q_2$ are the input differential pair.

$Q_3$ must also be considered now because its input resistance forms part of $Q_1$'s collector load resistance. If the input signal is regarded as $v^+$ with respect to ground, $Q_2$ looks like a common base connection and can be represented by its common base input resistance $1/g_{m2}$. The collector current of $Q_1$ sees two resistors in parallel, $R_1$ and the input resistance of $Q_3$. $Q_3$ is a common emitter amplifier *without* degeneration. Its input resistance is $r_{be3}$.



A small signal equivalent circuit describes the three transistor circuit block according to our simplifications.

This small signal model is very similar to the common emitter *with* degeneration from Lecture 1. In this case $R_S = 0$ and $R_E$ and $R_L$ are parallel combinations $R_E // r_{e2}$ and $R_1 // r_{be3}$. Since $R_E >> r_{e2}$, $r_{e2}$ dominates. The gain expression for the circuit is (based on the degenerated CE analysis)

$$\frac{v_{o1}}{v_i} = -\frac{R_1 // r_{be3}}{r_{e1} + r_{e2}} \quad (27)$$

# Review

- Considered the emitter follower circuit (voltage gain, current gain, input and output resistances).
- Considered the common base circuit (voltage gain, current gain, input resistance).
- Recapped the idea of the opamp as a feedback system.
- Introduced a simplified schematic of an opamp.
- Developed some ideas around the DC conditions of the simplified opamp
- Looked at the combination of three transistors into a differential amplifier + common emitter stage and considered their combined effect.