First Order Circuits

1 Introduction

There are two forms of first order frequency response, low-pass and high-pass. There is also a third hybrid form which is a linear sum of the low-pass and high-pass cases; this is not really a separate type of response in its own right. The terms high-pass and low-pass relate to the way the circuit gain changes as frequency is varied. A low-pass circuit tends to pass frequencies below a critical value but attenuates increasingly as frequency exceeds this critical value. The high-pass circuit, on the other hand, passes frequencies above some critical value and attenuates increasingly as frequency falls below this critical value. The critical value is usually called the **corner frequency** or the **3dB frequency**. In working out transfer functions it is important to keep j and ω together and replacing $j\omega$ with s is a convenient way of achieving this objective. s is actually the Laplace complex frequency variable which reduces to $j\omega$ for steady state frequency response considerations.

The two forms of first order response can be represented by standard forms and although the hybrid form is a sum of low-pass and high-pass, it is usually convenient to treat it as a third standard form. All first order circuits can be interpreted by forcing their transfer functions into the shape of a standard form and then extracting the relevant parameters by inspection.

2 First order standard forms

A general transfer function will have a **denominator** of the form $a_0 + a_1 s + a_2 s^2 + a_3 s^3 + ..$. A transfer function is first order if only the a_0 and $a_1 s$ terms exist. From a frequency response point of view, $s \Rightarrow j\omega$.

The two basic forms of first order transfer function are;

The low-pass

$$\frac{v_o}{v_i} = k \cdot \frac{1}{1 + s\tau} = k \cdot \frac{1}{1 + j\dfrac{\omega}{\omega_o}} = k \cdot \frac{1}{1 + j\dfrac{f}{f_o}} \tag{2.1}$$

The high-pass

$$\frac{v_o}{v_i} = k \cdot \frac{s\tau}{1 + s\tau} = k \cdot \frac{j\dfrac{\omega}{\omega_o}}{1 + j\dfrac{\omega}{\omega_o}} = k \cdot \frac{j\dfrac{f}{f_o}}{1 + j\dfrac{f}{f_o}} \tag{2.2}$$

The third form, which is a linear sum of **(2.1)** and **(2.2)**, is often called a "pole-zero" or "lead lag" function.

$$\frac{v_o}{v_i} = k \cdot \frac{1 + s\tau_1}{1 + s\tau_2} = k \cdot \frac{1 + j\dfrac{\omega}{\omega_1}}{1 + j\dfrac{\omega}{\omega_2}} = k \cdot \frac{1 + j\dfrac{f}{f_1}}{1 + j\dfrac{f}{f_2}} \tag{2.3}$$

The key points about these standard forms are

(i) The denominator is always complex

(ii) Whatever multiplies $j\omega$ **in the denominator** is the system time constant. In frequency domain expressions it is very common to see time constant expressed in terms of a frequency domain constant as in **(2.1)**, **(2.2)** and **(2.3)**.

(ii) The denominator has a real part of unity in all cases.

(iii) The numerator may be real (constant) as in **(2.1)**, imaginary as in **(2.2)** or complex as in **(2.3)**.

(iv) Where the numerator is purely imaginary, the coeficcient of $j\omega$ in the top line should be made to be the same as that of the imaginary part of the denomiator.

(v) Where the numerator is complex, its real part should be forced to unity.

(vi) The form of the numerator indicates the type of first order response -

purely real	⇒	low-pass (or simple integrator)
purely imaginary	⇒	high-pass (or simple differentiator)
complex	⇒	"pole-zero" or "lead-lag" circuit - a linear sum of low-pass and high-pass, each with different *k*.

3 Getting the transfer function

The transfer function will often be a suitably manipulated potential divider relationship. In order to end up with a result that is easily interpretable, it is desirable to express the transfer function in a particular way. An outline of the steps necessary is as follows with the circuit of figure 1 used as an example.

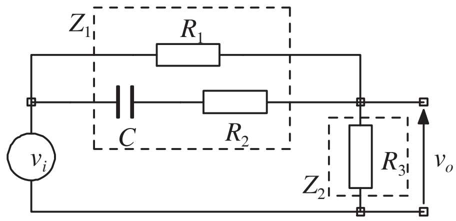


Figure 1

An example first order RC circuit

(i) Work out the impedances Z_1 and Z_2 remembering to keep ($j\omega$) together. For figure 1 and using s for $j\omega$ they are

$$Z_1 = R_1 \,//\, (R_2 + X_C)$$

$$= \frac{R_1\left(R_2 + \dfrac{1}{sC}\right)}{R_1 + R_2 + \dfrac{1}{sC}} = \frac{R_1(1 + sCR_2)}{1 + sC(R_1 + R_2)} \qquad \textbf{(3.1)}$$

$$Z_2 = R_3 \qquad \textbf{(3.2)}$$

(ii) Write down the potential division relationship. For the circuit of figure 1 it is

$$\frac{v_o}{v_i} = \frac{Z_2}{Z_2 + Z_1} = \frac{R_3}{R_3 + \dfrac{R_1\,(1 + sCR_2)}{1 + sC\,(R_1 + R_2)}} \qquad \textbf{(3.3)}$$

(iii) Manipulate the potential division relationship to end up with a ratio of two polynomials in s. Note that in general the numerator polynomial may be completely real, completely imaginary or complex; the denominator will be complex with a real and an s term. For the circuit of figure 1,

$$\frac{v_o}{v_i} = \frac{R_3\,(1 + sC\,(R_1 + R_2))}{R_3\,(1 + sC\,(R_1 + R_2)) + R_1\,(1 + sCR_2)}$$

$$= \frac{R_3\,(1 + sC\,(R_1 + R_2))}{R_3 + R_1 + sC\,(R_1R_3 + R_2R_3 + R_1R_2)} \qquad \textbf{(3.4)}$$

(iv) Take out factors to force the real parts of the numerator and denominator to unity. This will often result in having to divide the *s* term in the denominator by the real part of the denominator. The numerator often naturally occurs in the right form (as in this example). For the circuit of figure 1 R_3 is obviously a factor in the numerator. $(R_1 + R_3)$ is the factor that must be removed from the denominator to give a denominator real part of unity. These two factors form a dimensionless frequency independent ratio that multiplies the complex part of the expression.

$$\frac{v_o}{v_i} = \frac{R_3}{R_1 + R_3} \cdot \frac{1 + sC\,(R_1 + R_2)}{1 + sC\,\dfrac{R_1R_2 + R_2R_3 + R_1R_3}{R_1 + R_3}} \qquad \textbf{(3.5)}$$

At each stage of this process you should get into the habit of checking that your equations are dimensionally consistent. It is easy to check dimensions and although dimensional checks will not reveal all errors, they will reveal a significant number.

4 Interpreting the transfer function

Having obtained the transfer function and manipulated it so that it has the shape of a standard form, the next step is to compare the transfer function with the standard form of the same type. Again, using the circuit of figure 1 as an example and comparing **(3.5)** with **(2.1)**, **(2.2)** and **(2.3)**, it is clear that **(3.5)** is of the form of **(2.3)** - the hybrid form - and by comparison of coefficients,

$$k = \frac{R_3}{R_1 + R_3},\ \omega_1 = \frac{1}{C\,(R_1 + R_2)} \text{ and } \omega_2 = \frac{R_1 + R_3}{C\,(R_1R_2 + R_1R_3 + R_2R_3)} \qquad \textbf{(4.1)}$$

Knowledge of these three parameters and the the type of response ((2.1), (2.2), or (2.3)) specifies the shape of the amplitude and phase responses of the circuit as shown in section 5. It is also possible to use the transfer function to identify system gain as frequency approaches very low or very high values - the low frequency gain and the high frequency gain. To do this one must consider how the modulus of the transfer function behaves as frequency becomes very small or very large. Taking the hybrid standard form of **(2.3)**,

$$\left|\frac{v_o}{v_i}\right| = k\,.\left|\frac{1 + j\dfrac{\omega}{\omega_1}}{1 + j\dfrac{\omega}{\omega_2}}\right| = k\,.\left(\frac{1 + \dfrac{\omega^2}{\omega_1^2}}{1 + \dfrac{\omega^2}{\omega_2^2}}\right)^{\frac{1}{2}} \qquad \textbf{(4.2)}$$

At low frequencies, $\omega << \omega_1$ and $\omega << \omega_2$ so both $\dfrac{\omega^2}{\omega_1^2}$ and $\dfrac{\omega^2}{\omega_2^2}$ are << 1 and $\left|\dfrac{v_o}{v_i}\right| \approx k$. **(4.3)**

At high frequencies, $\omega >> \omega_1$ and $\omega >> \omega_2$ so both $\dfrac{\omega^2}{\omega_1^2}$ and $\dfrac{\omega^2}{\omega_2^2}$ are >> 1 and $\left|\dfrac{v_o}{v_i}\right| \approx k\,\dfrac{\omega_2}{\omega_1}$. **(4.4)**

5 Response shapes

There are three response shapes that correspond to the three standard forms of **(2.1)**, **(2.2)** and **(2.3)**. **All** first order transfer functions will fall into one of these three categories. Amplitude responses are usually plotted with gain in dB; phase is usually plotted on a linear scale. Both amplitude and phase are usually plotted with a logarithmic frequency axis.

5.1 Low-Pass

(a) Amplitude response

The low-pass amplitude response shape can be worked out by considering the modulus of **(2.1)** for frequencies well below, well above and in the region of, ω_0.

$$\left|\frac{v_o}{v_i}\right| = k \,.\, \left|\frac{1}{1 + j\dfrac{\omega}{\omega_0}}\right| = k \,.\, \left(\frac{1}{1 + \dfrac{\omega^2}{\omega_0^2}}\right)^{\frac{1}{2}}$$

(i) $\omega \ll \omega_0$

Under this condition $\dfrac{\omega^2}{\omega_0^2}$ is much smaller than unity so $\left|\dfrac{v_o}{v_i}\right| \approx k$. ($\equiv 20\log k$ dB)

(ii) $\omega = \omega_0$

Under this condition $\dfrac{\omega^2}{\omega_0^2} = 1$ so $\left|\dfrac{v_o}{v_i}\right| \approx \dfrac{k}{\sqrt{2}}$. ($\equiv 20\log k$ dB − 3dB)

(iii) $\omega \gg \omega_0$

Under this condition $\dfrac{\omega^2}{\omega_0^2}$ is much larger than unity so $\left|\dfrac{v_o}{v_i}\right| \approx k\dfrac{\omega_0}{\omega}$. Thus the circuit gain is inversely proportional to frequency; if ω increases by a factor of 10, gain decreases by a factor of 10. A factor of 10 reduction in gain is a reduction of 20 dB so the slope of the amplitude response in this frequency region will approach −20dB for every decade increase in frequency.

A good approximation to the amplitude response (known as the Bode approximation) draws the response as two straight lines - a horizontal line at the low frequency gain from 0Hz to ω_0 and a -20dB per decade line from ω_0 upwards. The low-pass amplitude response is shown in figure 2.

(b) Phase response

The phase of the low-pass response of **(2.1)** is calculated from $\varphi = -\tan^{-1}\dfrac{\omega}{\omega_0}$ and as in the amplitude case, its shape can be deduced by considering three frequency conditions,

(i) $\omega \ll \omega_0$

Under this condition, as $\omega \Rightarrow 0$, $\varphi \Rightarrow 0^o$.

(ii) $\omega = \omega_0$

Under this condition, $\varphi = -\tan^{-1}\dfrac{\omega}{\omega_0} = -\tan^{-1} 1 = -45^o$

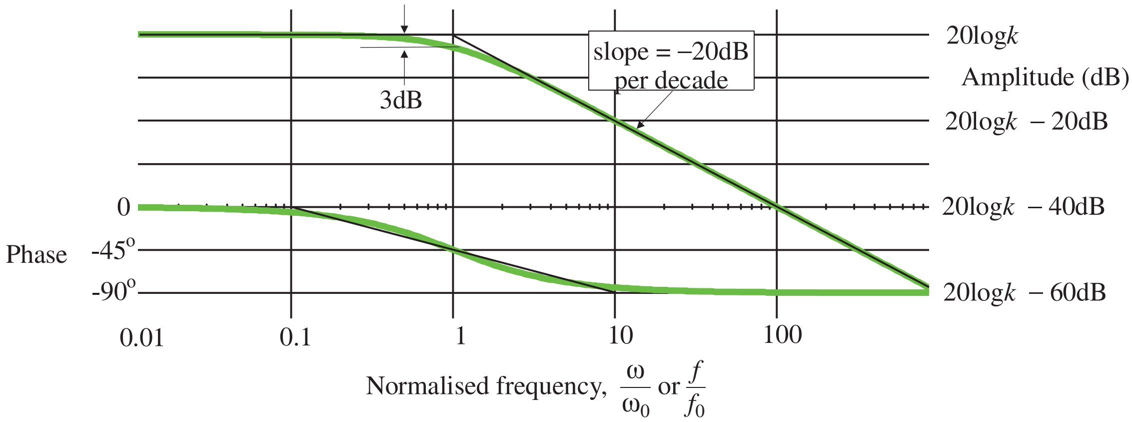
(iii) $\omega >> \omega_0$

Under this condition, $\varphi = -\tan^{-1}\dfrac{\omega}{\omega_0} \Rightarrow -\tan^{-1}$ [a large number] $\Rightarrow -90^o$

The Bode approximation for the phase response is a straight line starting from 0^o at $\omega = 0.1\omega_0$, going through -45^o at $\omega = \omega_0$ and reaching -90^o at $\omega = 10\,\omega_0$. Its slope is therefore -45^o per decade. The phase response is shown in figure 2.

Figure 2

Low-pass amplitude and phase response



5.2 High-Pass

(a) Amplitude response

The high-pass transfer function of **(2.2)** has a magnitude response that is a mirror image of the low-pass response about the vertical line $\omega/\omega_0 = 1$. The response plot for high-pass function can be written as

$$20\log\left|\frac{v_o}{v_i}\right| = 20\log\left(k\,.\left|\frac{j\dfrac{\omega}{\omega_0}}{1+j\dfrac{\omega}{\omega_0}}\right|\right) = 20\log k + 20\log\left|j\frac{\omega}{\omega_0}\right| + 20\log\left|\frac{1}{1+j\dfrac{\omega}{\omega_0}}\right|$$

and this makes it clear that on a logarithmic amplitude plot, the response consists of a sum of three components.

$20\log k$ is a constant.

$20\log\left|j\dfrac{\omega}{\omega_0}\right|$ is a straight line with a slope of +20 dB per decade that goes through 0 dB when $\omega = \omega_0$

$20\log\left|\dfrac{1}{1+j\dfrac{\omega}{\omega_0}}\right|$ is the low pass response shape, without the k, considered in section 5.1.

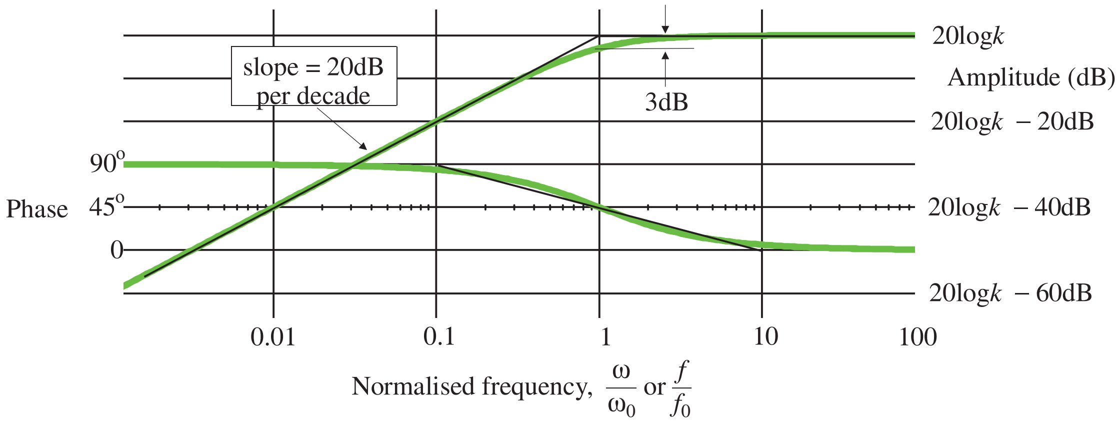
The high-pass amplitude response is shown in figure 3.

(b) Phase response

The phase response of the high-pass function is the same shape as that of the low-pass function but at all frequencies the high-pass phase is 90^o higher than the low-pass phase. The difference arises because there is a *j* term but no real term in the numerator and that *j* term acts as a 90^o phase shift operator. The phase response of the high-pass function is shown in figure 3.

Figure 3

High- pass amplitude and phase response



5.3 Pole-Zero or Lead-Lag

(a) Amplitude response

Using the same approach as in section 5.2, the log of the modulus of **(2.3)** can be expressed as the sum of simpler logarithmic components

$$20\log\left|\frac{v_o}{v_i}\right| = 20\log\left(k\,.\left|\frac{1+j\dfrac{\omega}{\omega_1}}{1+j\dfrac{\omega}{\omega_2}}\right|\right) \quad \text{or}$$

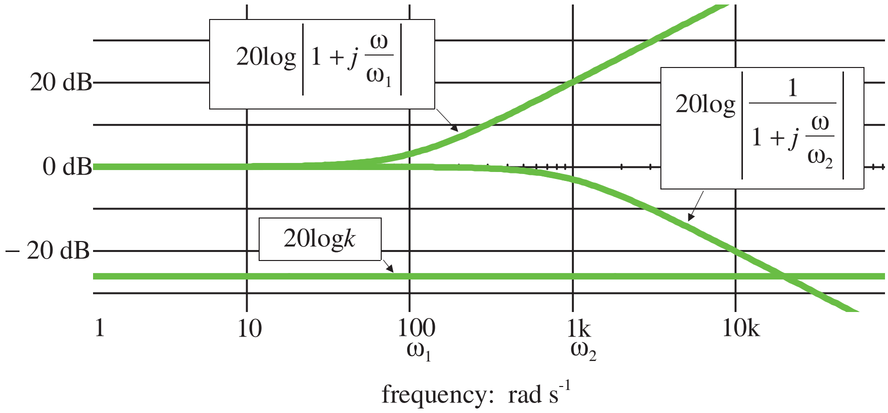
$$20\log\left|\frac{v_o}{v_i}\right| = 20\log k + 20\log\left|1+j\frac{\omega}{\omega_1}\right| + 20\log\left|\frac{1}{1+j\dfrac{\omega}{\omega_2}}\right| \qquad \textbf{(5.1)}$$

The only new part here is the second term. Since $20\log\left|1+j\frac{\omega}{\omega_1}\right| = -20\log\left|\frac{1}{1+j\frac{\omega}{\omega_1}}\right|$, the response of the second term is the inverse of the first order low pass response of section 5.1. In other words for the second term, the gain is 0 dB at 0 Hz, rises to + 3 dB at $\omega = \omega_1$ and rises at 20 dB per decade for frequencies greater than ω_1.

If $\omega_1 < \omega_2$, the overall gain rises as frequency increases between ω_1 and ω_2 before flattening off when $\omega > \omega_2$. If $\omega_1 > \omega_2$, the overall gain falls as frequency increases between ω_2 and ω_1 before flattening off when $\omega > \omega_1$. Figure 4a shows the gain components of **(5.1)** and figure 4c shows the overall sum of those components, together with the overall phase response.

Figure 4a

The three log magnitude components of **(5.1)**. *In this example, ω_1 is 100 rad s^{-1} and ω_2 is 1000 rad s^{-1}.*



(b) Phase response

For a function such as **(2.3)** the phase is given by

$$\varphi = \tan^{-1}\frac{\omega}{\omega_1} - \tan^{-1}\frac{\omega}{\omega_2} \qquad \textbf{(5.2)}$$

There is no phase shift associated with the constant *k*. Both parts of the phase expression have a phase that approaches 0° at low frequencies and approaches 90° for high frequencies. Since the two subtract, the high frequency phase shift will also be zero. In the region of ω_1 and ω_2, the phase will be a positive going or negative going hump depending upon whether $\omega_1 < \omega_2$ or vice versa. Figure 4b shows the contribution to phase made by each of the components of **(5.2)** and figure 4c shows the sum of these components.

Figure 4b

The three phase components of **(5.2)**. *In this example, ω_1 is 100 rad s^{-1} and ω_2 is 1000 rad s^{-1}.*

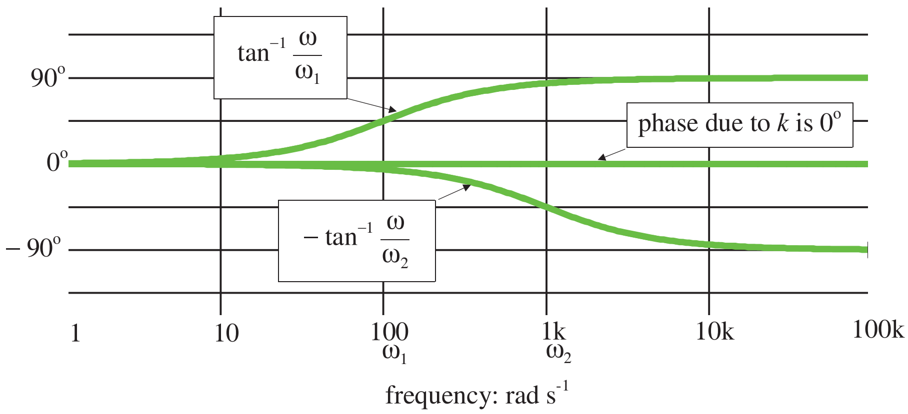
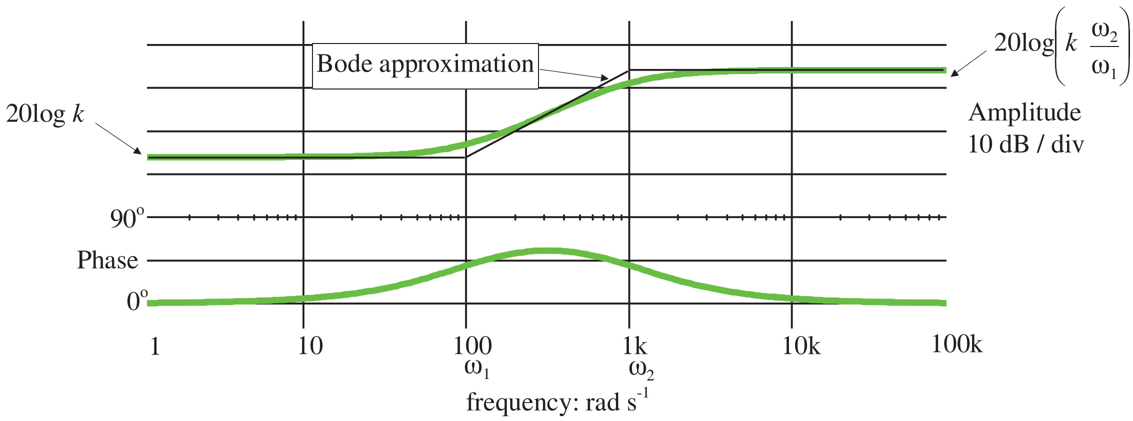


Figure 4c

Overall response of a pole-zero circuit such as **(2.3)** *with* $\omega_1 = 100$ *rad* s^{-1} *and* $\omega_2 = 1000$ *rad* s^{-1}*. The amplitude response is the sum of the components of shown in figure 4a and the phase response is the sum of the components shown in figure 4b.*



Note that in general ω_1 can be smaller than or larger than ω_2. The response shown here is for ω_1 smaller than ω_2. If ω_2 had been smaller than ω_1, gain and phase would have started falling because of the effects of ω_2 before they flattened out because of the effects of ω_1. The phase response would then be a downwards going hump and the amplitude response would have a higher value at low frequencies than at high frequencies.

For the circuit of figure 1, ω_1 is lower than ω_2 for all possible component value combinations.

6 Checking by inspection

It is quite easy to identify high frequency gain, low frequency gain and time constant by inspection. Identifying these parameters is a useful check on the accuracy of your algebraic manipulations; if the time constant is different depending on how you calculated it, there is an error somewhere. The following section outlines the steps in the checking process using the circuit of figure 1 as an example.

(i) Low frequency (l.f.) gain

The low frequency gain is the gain that is approached as $f \Rightarrow 0$. To work it out, replace capacitors with a very high impedance. Capacitors then dominate the impedance of series *RC* combinations but are of negligible effect in parallel *RC* combinations. In most cases the capacitors are simply removed from the circuit. Thus the low frequency equivalent circuit of figure 1 is given in figure 5 and the gain is easily written down as

$$\frac{v_o}{v_i} = \frac{R_3}{R_1 + R_3}$$

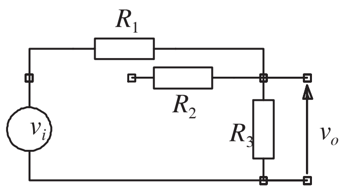


Figure 5

The low-frequency equivalent circuit of figure 1. Note that C has been replaced by an open circuit.

(ii) High frequency (h.f.) gain

The high frequency gain is the gain that is approached as $f \Rightarrow \infty$. In this case capacitors have a very low reactance so the impedance of series *RC* combinations is dominated by *R* and that of parallel combinations is dominated by *C*. The high frequency equivalent circuit of figure 1 is shown in figure 6. Again, the gain can be easily written down as

$$\frac{v_o}{v_i} = \frac{R_3}{R_1//R_2 + R_3} = \frac{R_3\,(R_1 + R_2)}{R_1R_2 + R_1R_3 + R_2R_3}$$

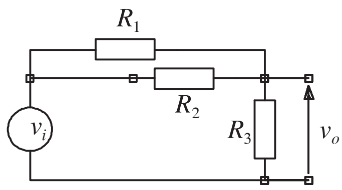


Figure 6

A high frequency equivalent circuit of figure 1. Note that C has been replaced by a short circuit.

(iii) Time constant

To identify the system time constant one must look at the circuit from the capacitor's point of view. First replace all sources by their Thevenin equivalent impedances - 0Ω for a voltage source and ∞Ω for a current source. Then imagine that you can inject some charge into the capacitor and ask yourself what is the resistance of the discharge path. C multiplied by the discharge path resistance is the system time constant and this should be the same as the coefficient of $j\omega$ in the denominator of the transfer function.

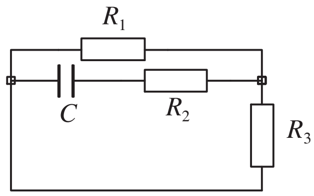


Figure 7

Equivalent circuit of figure 1 for identifying time constant

Figure 7 shows figure 1 with v_i replaced by a short circuit. Charge in C must flow through R_2. After passing through R_2, the current is faced with R_1 and R_3 in parallel giving a time constant

$$\tau = C\,(R_2 + R_1//R_3) = C\,\frac{R_1R_2 + R_1R_3 + R_2R_3}{R_1 + R_3}$$

7 Step response

A step input is an instantaneous change in input voltage from one voltage to another. The instant at which the change occurs is usually taken as $t = 0$ although it doesn't have to be there. A unit step input is a change from 0V to 1V.

Step inputs are very useful test signals because many circuit applications deal with signals that change state suddenly from one value to another. The step response of a circuit, ie the output that arises as a result of a step at the input is therefore a useful response to be able to predict.

For first order circuits, the step response will in general consist of a step followed by an exponential. The magnitude of the step can be calcuated from the gain terms and the exponential can be written

$$V(t) = (V_{START} - V_{FINISH})\, e^{-t/\tau} + V_{FINISH} \qquad \textbf{(7.1)}$$

The numbers needed to define V_{START}, V_{FINISH} and τ can be found from the input step magnitude, the low frequency ($f \Rightarrow 0$) gain, the high frequency ($f \Rightarrow \infty$) gain and the system time constant - all these can be found by inspection as described in section 6 and they apply here as follows.

- The high frequency gain operates on the instantaneous step
- The low frequency gain operates on the dc voltage that exists before the step occurs - this is often 0V - and defines the voltage that will be reached as $t \Rightarrow \infty$

As an example, consider the circuit of figure 1, redrawn for convenience as figure 8 with component values added and a step input going from −2 V to + 8 V at $t = 0$.

The low frequency gain of the circuit is

$$A_{LF} = \frac{1\text{k}\Omega}{1\text{k}\Omega + 10\text{k}\Omega} = 90.9 \times 10^{-3}$$

This defines the voltage from which any step on the output begins - in this case it is

$$-2\text{V} \times 90.9 \times 10^{-3} = -0.18\text{V}$$

and also the voltage aimed for as $t \Rightarrow \infty$ which is

$$8\text{V} \times 90.9 \times 10^{-3} = 0.73\text{V}$$

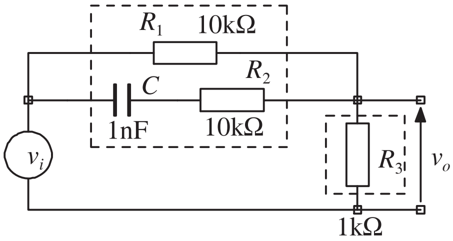


Figure 8

The first order RC circuit of figure 1 with values added. v_i is a step defined by $v_i = -2V$ for $t < 0$ and $v_i = 8V$ for $t > 0$

The high frequency gain of the circuit is

$$A_{HF} = \frac{1\text{k}\Omega}{10\text{k}\Omega // 10\text{k}\Omega + 1\text{k}\Omega} = 0.167$$

and this, when multiplied by the height of the input step defines the height of the output step as

$$0.167 \times (8\text{V} - (-2\text{V})) = 1.67\text{V}$$

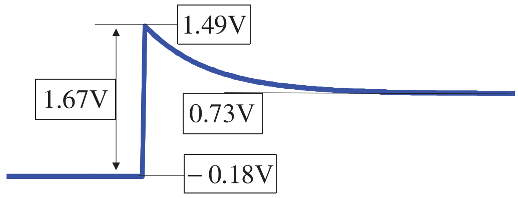


Figure 9

The step response associated with figure 8

The overall response is shown in figure 9 with the key voltages labelled. The exponential decay has $V_{START} = 1.49\text{V}$, $V_{FINISH} = 0.73\text{V}$ and $\tau = 10.9\mu\text{s}$ so using **(7.1)**, the exponential part of the response is $V(t) = (1.49 - 0.73)\, e^{-t/10.9 \times 10^{-6}} + 0.73$ **.**

8 An example

Identify the behaviour of the circuit of figure 10.

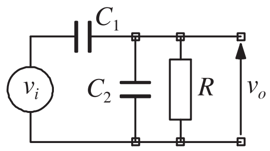


Figure 10

An example circuit

(i) By inspection

At low frequency the reactance of C_1 is much larger than the impedance of the C_2R combination so in the limit of $f \Rightarrow 0$, l.f. gain = 0

At high frequency the reactances of both capacitors are small compared with R and C_2 dominates the C_2R combination. As $f \Rightarrow \infty$, the gain is determined by the capacitive potential division between C_1 and C_2 so h.f. gain is

$$\frac{\dfrac{1}{j\omega C_2}}{\dfrac{1}{j\omega C_1} + \dfrac{1}{j\omega C_2}} = \frac{C_1}{C_1 + C_2} \qquad \textbf{(8.1)}$$

The time constant will be $R(C_1 // C_2)$ which is $R\,(C_1 + C_2)$ **(8.2)**

(ii) By analysis

The transfer function is a potential division between C_1 and the parallel combination C_2R.

$$\frac{v_o}{v_i} = \frac{\dfrac{\dfrac{R}{j\omega C_2}}{R + \dfrac{1}{j\omega C_2}}}{\dfrac{1}{j\omega C_1} + \dfrac{\dfrac{R}{j\omega C_2}}{R + \dfrac{1}{j\omega C_2}}} = \frac{\dfrac{R}{1 + j\omega C_2 R}}{\dfrac{1}{j\omega C_1} + \dfrac{R}{1 + j\omega C_2 R}} = \frac{j\omega C_1 R}{1 + j\omega C_2 R + j\omega C_1 R}$$

$$= \frac{j\omega C_1 R}{1 + j\omega(C_2 + C_1)R} = \frac{C_1}{C_1 + C_2} \cdot \frac{j\omega(C_1 + C_2)R}{1 + j\omega(C_2 + C_1)R} \equiv k \cdot \frac{j\dfrac{\omega}{\omega_c}}{1 + j\dfrac{\omega}{\omega_c}} \qquad \textbf{(8.3)}$$

The analysis of **(8.3)** has five steps. Step 1 is the raw potential divider expression that is successively simplified to step 4. Step 4 is clearly a high pass response because of the purely imaginary numerator but the standard form of **(2.2)** (repeated as a sixth term in **(8.3)**) requires that the coefficient of $j\omega$ in the numerator is forced to that in the denominator. This can be easily achieved at the expense of introducing a constant multiplier term consisting here of a capacitive potential divider. The l.f. gain, h.f. gain and time constant obtained from **(8.3)** are consistent with those obtained by inspection in section 6.

(iii) Step response

Assume an input step from 0 V to V_1 V at $t = 0$

The l.f. gain $\Rightarrow 0$ as $\omega \Rightarrow 0$ rad s^{-1} so as $t \Rightarrow \infty$, $v_o \Rightarrow 0$.

The h.f. gain $\Rightarrow \dfrac{C_1}{C_1 + C_2}$ as $f \Rightarrow \infty$ so the step size is $V_1 \times \dfrac{C_1}{C_1 + C_2}$

The output waveshape is therefore a voltage step from 0 V to $V_1 \times \dfrac{C_1}{C_1 + C_2}$ V followed by an exponential of the form $V(t) = \left(V_1 \times \dfrac{C_1}{C_1 + C_2}\right) e^{-t/\tau}$ where $\tau = R\,(C_1 + C_2)$ **.**

Remember that the start voltage for the exponential is always the voltage at the end of any step arising at the output because of the transient change of input. Low-pass transfer functions (i.e., those of the same shape as **(2.1)**) do not have an output step in their step response whereas high-pass and pole-zero responses (i.e, **(2.2)** and **(2.3)**) always do.

9 Concluding comments

All the discussion here has been in terms of passive *RC* circuits. First order behaviour is also exhibited by *LR* circuits and by active circuits such as op-amp based amplifiers. The three standard forms apply to all manifestations of first order frequency dependent behaviour.

RCT 9/07