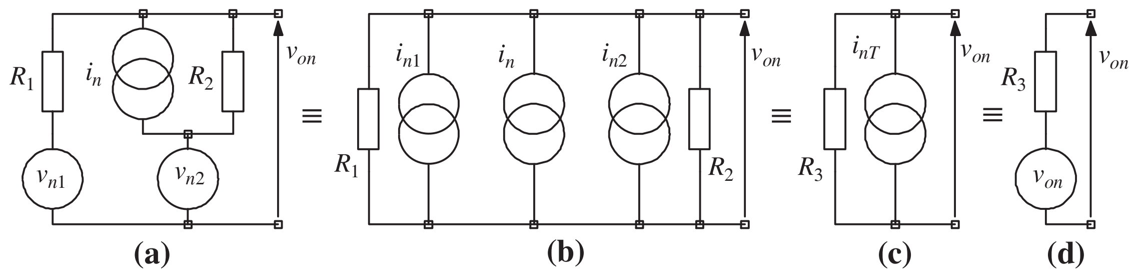
The University of Sheffield, Department EEE

Analogue and Digital Electronics
Problem Sheet Solutions: Noise

Q1 (i) There are several ways in which a problem like this can be solved, all based on standard circuit analysis methods. One approach, illustrated in figure 7, uses Thevenin to Norton transformations to simplify the circuit until the problem becomes trivial. The circuit of figure 7a can be transformed into figure 7b by performing Thevenin to Norton transformations:



Figures 1 a, b, c and d

v_{n1} in series with $R_1 \Rightarrow i_{n1}$ in parallel with R_1, where $i_{n1} = v_{n1}/R_1$, and

v_{n2} in series with $R_2 \Rightarrow i_{n2}$ in parallel with R_2, where $i_{n2} = v_{n2}/R_2$.

The second of these is not quite as obvious as the first unless it is appreciated that it makes no difference to the circuit whether the bottom end of i_n is connected to the top or the bottom of v_{n2}. This can be verified easily by confirming that the component of output voltage due to i_n is independent of v_{n2}. The three current sources and two resistors of figure 7b can be combined into the single current source i_{nT} in parallel with the single resistor R_3, shown in figure 7c, where $\overline{i_{nT}^2} = \overline{i_{n1}^2} + \overline{i_{n2}^2} + \overline{i_n^2}$ and $R_3 = R_1//R_2$, and this can in turn be transformed into the Thevenin form of figure 7d where:

$$R_3 = R_1//R_2 = 12\text{k}\Omega//22\text{k}\Omega = 7.76\ \text{k}\Omega \text{ and}$$

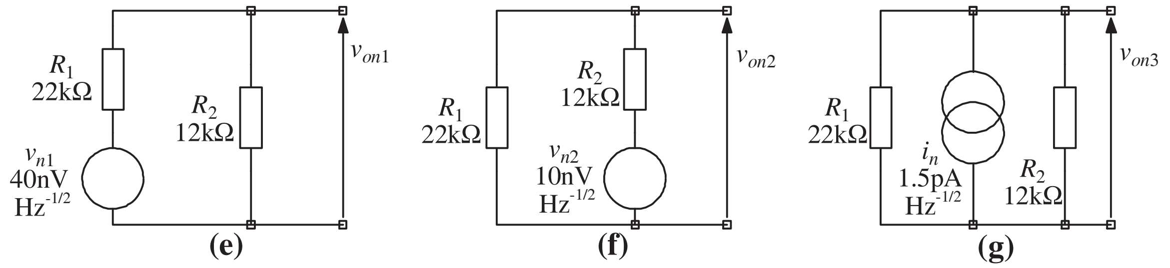
$$\overline{v_{on}^2} = \overline{i_{nT}^2}R_3^2 = R_3^2\left(\frac{\overline{v_{n1}^2}}{R_1^2} + \frac{\overline{v_{n2}^2}}{R_2^2} + \overline{i_n^2}\right) = (7.76\ \text{k}\Omega)^2\left(\frac{(40\text{nV})^2}{(22\text{k}\Omega)^2} + \frac{(10\text{nV})^2}{(12\text{k}\Omega)^2} + (1.5\text{pA})^2\right)$$

$$= 376.8 \times 10^{-18}\ \text{V}^2\ \text{Hz}^{-1} = 19.4\ \text{nV Hz}^{-1/2}$$

A second approach is based on superposition. Figures 7e, 7f and 7g show the three partial circuits that define the component of output voltage due to each of the three generators.

$$\overline{v_{on1}^2} = \overline{v_{n1}^2}\frac{R_2^2}{(R_1 + R_2)^2},\quad \overline{v_{on2}^2} = \overline{v_{n2}^2}\frac{R_1^2}{(R_1 + R_2)^2} \text{ and } \overline{v_{on3}^2} = \overline{i_n^2}\frac{(R_1R_2)^2}{(R_1 + R_2)^2}$$

$\overline{v_{on}^2} = \overline{v_{on1}^2} + \overline{v_{on2}^2} + \overline{v_{on3}^2}$ which gives the same answer as the Thevenin approach above.



Figures 1 e, f and g

(ii) To find the total rms noise voltage over a defined bandwidth, the spectral density must be integrated over the bandwidth of interest, ie

$$\overline{v_{nT}^2} = \overline{v_{on}^2}\,\Delta f = 376.8 \times 10^{-18} \times 20\text{ kHz} = 7.54 \times 10^{-12}\text{ V}^2$$

or $v_{onT} = 2.75\ \mu\text{V}$

(iii) The temperature to which R_3 must be notionally raised in order to generate the same mean squared noise voltage as the source v_{on} is given by:

$$\overline{v_{on}^2} = 4kT_E R_3 \text{ or } T_E = \text{Noise temp.} = \frac{\overline{v_{on}^2}}{4kR_3} = \frac{376.8 \times 10^{-18}}{4 \times 1.38 \times 10^{-23} \times 7760} = 880\text{ K.}$$

Q2 This question defines the noise sources in a number of ways. In figure 2 R_S has been replaced by its noise equivalent circuit where $\overline{v_{ns}^2} = 4kTR_S\ \text{V}^2\,\text{Hz}^{-1}$. Since the circuit is a series circuit containing a current source, the current in the series circuit is dependent only on the current source. Taking a superposition approach

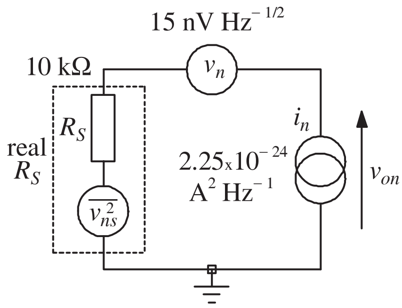


Figure 2

$$\overline{v_{on}^2} \text{ due to } R_S = \overline{v_{ns}^2} = 4kTR_S = 166 \times 10^{-18}\text{ V}^2\,\text{Hz}^{-1}$$

$$\overline{v_{on}^2} \text{ due to } v_n = \overline{v_{on}^2} = 225 \times 10^{-18}\text{ V}^2\,\text{Hz}^{-1}$$

$$\overline{v_{on}^2} \text{ due to } \overline{i_n^2} = \overline{i_n^2}R_S^2 = 225 \times 10^{-18}\text{ V}^2\,\text{Hz}^{-1}$$

$$\text{total } \overline{v_{on}^2} = [166 + 225 + 225] \times 10^{-18} = 616 \times 10^{-18}\text{ V}^2\,\text{Hz}^{-1}$$

$$\text{thus } v_{on} = 24.8\text{ nV Hz}^{-1/2}$$

Q3 (i) In order to work out the magnitude of the shot noise generated by the diode, the dc current flow through it must be evaluated. Assuming a diode voltage drop of 0.7V, $I_D = (20 - 0.7)/68\text{k}\Omega = 284\mu\text{A}$ and the mean squared shot noise current is $2eI_D = 91.0 \times 10^{-24}\text{ A}^2\,\text{Hz}^{-1}$. The thermal noise associated with the resistor is $4kTR = 1.126 \times 10^{-15}\text{ V}^2\,\text{Hz}^{-1}$.

The noise equivalent circuit contains only noise sources so the 20V dc source is not included. The resistor is represented by its thermal noise voltage generator in series with an ideal resistor and the diode is represented by the diode incremental (or "slope" or "differential" or "small signal") resistance in parallel with the shot noise current generator. Diode incremental resistance is given by $kT/eI_D = 91\Omega$ in this case. The noise equivalent circuit is shown in figure 3.

Evaluation of output noise is simply a matter of dealing with this circuit:

$$\overline{v_{on}^2} = \overline{i_{nd}^2}\frac{R^2 r_D^2}{(R+r_D)^2} + \overline{v_{nR}^2}\frac{r_D^2}{(R+r_D)^2} = \frac{r_D^2}{(R+r_D)^2}(\overline{i_{nd}^2}R^2 + \overline{v_{nR}^2}) = 754 \times 10^{-21}\ \text{V}^2\ \text{Hz}^{-1}$$

or $v_{on} = 868\ \text{pV Hz}^{-1/2}$

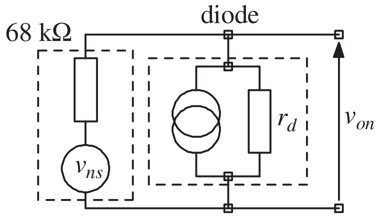


Figure 3

(ii) By inspection, the Thevenin equivalent resistance from which this noise voltage appears to come is $R\ //\ r_D = 68\text{k}\Omega\ //\ 91\Omega \approx 91\ \Omega$.

(iii) The temperature at which a 91Ω resistor would have to be maintained in order to generate the noise voltage calculated in **(i)** is given by;

$$754 \times 10^{-21} = 4kT_E\, 91 \text{ or } T_E = 150\ \text{K}$$

(iv) If the output is loaded by a capacitor, the total noise voltage at the output will be given by the kT/C noise at the noise temperature calculated in **(iii)**, ie

$$\overline{v_{onT}^2} = \frac{kT_E}{C} = 207 \times 10^{-12}\ \text{V}^2 \text{ or } v_{onT} = 14.4\ \mu\text{V}$$

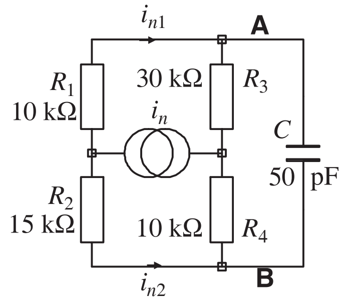


Figure 4

Q4 There are four steps in the solution of this problem

- reduce the circuit to the left of **AB** to a Thevenin equivalent of R_{Th} in series with v_{Th}
- find the noise temperature, T_E, of R_{Th}
- use kT_E/C to find total mean squared noise voltage across C
- square root this to get total rms v_{nc}

First deal with i_n. Consider an instant when i_n is flowing towards R_1 and R_2. The current splitting rule will give an i_{n1} and i_{n2} of

$$i_{n1} = \frac{i_n(R_2+R_4)}{R_1+R_2+R_3+R_4} \text{ and } i_{n2} = \frac{i_n(R_1+R_3)}{R_1+R_2+R_3+R_4}$$

The voltages across R_3 and R_4 with a positive direction taken as upwards are

$$v_{R3} = i_{n1}R_3 = \frac{i_n R_3(R_2+R_4)}{R_1+R_2+R_3+R_4} \text{ and } v_{R4} = -i_{n2}R_4 = -\frac{i_n R_4(R_1+R_3)}{R_1+R_2+R_3+R_4}$$

$$v_{AB} \text{ due to } i_n \text{ is } v_3 + v_4 = \frac{i_n(R_2R_3 - R_1R_4)}{R_1+R_2+R_3+R_4} = 32.3\ \text{nV Hz}^{-1/2} \text{ or } 1.04\times10^{-15}\ \text{V}^2\ \text{Hz}^{-1}$$

The Thevenin equivalent resistance looking into the terminals **AB** is $(R_3 + R_4)//(R_1 + R_2)$. This is 40 kΩ // 25 kΩ = 15.4 kΩ. The thermal noise associated with R_{Th}, which includes the noise generated by all its component resistors is $4kT\ 15.4 \times 10^3 = 255 \times 10^{-18}\ \mathrm{V^2\ Hz^{-1}}$.

The total mean sqared noise voltage in the Thevenin equivalent is the sum of that due to i_n and that due to R_{Th} so

$$\overline{v_{Th}^2} = (1040 + 255) \times 10^{-18} = 1.30 \times 10^{-15}\ \mathrm{V^2\ Hz^{-1}}$$

The noise temperature, T_E, of R_{Th} is the temprature to which R_{Th} would have to be raised in order for its thermal noise to account for the sum of noise due to i_n and room temperature thermal noise expected from R_{Th} so

$$4kT_E R_{Th} = 1.30 \times 10^{-15}\ \mathrm{V^2\ Hz^{-1}} \quad \text{or} \quad T_E = 1530\ \mathrm{K}$$

The total mean squared noise voltage across C is given by

$$\overline{v_{nc}^2} = \frac{kT_E}{C} = 422 \times 10^{-12}\ \mathrm{V^2} \quad \text{or} \quad v_{nc}$$

$= \mathbf{20.5\ \mu V}$

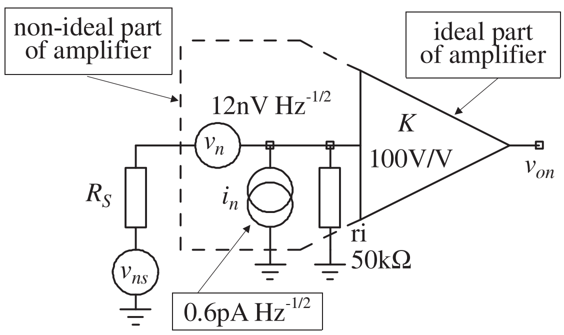


Figure 5

Q5 The noise equivalent circuit of the amplifier is as shown in figure 5. The resistance r_i is noise free because as part of the amplifier its effects are included in v_n and i_n. To work out the noise factor of the circuit, the mean squared noise output voltage from the real amplifier must be divided by that from an ideal version of the amplifier.

The mean sqare noise voltage at the output of the real amplifier is given by:

$$\overline{v_{onr}^2} = K^2\left(\frac{\overline{v_n^2}\,r_i^2}{(r_i + R_S)^2} + \frac{\overline{v_{nS}^2}\,r_i^2}{(r_i + R_S)^2} + \frac{\overline{i_n^2}\,r_i^2 R_S^2}{(r_i + R_S)^2}\right) = K^2\frac{r_i^2}{(r_i + R_S)^2}\left(\overline{v_n^2} + \overline{v_{nS}^2} + \overline{i_n^2}\,R_S^2\right) \qquad \textbf{(5.1)}$$

while for the ideal amplifier, v_n and i_n are zero giving:

$$\overline{v_{oni}^2} = K^2\frac{r_i^2}{(r_i + R_S)^2}\,\overline{v_{nS}^2} \qquad \textbf{(5.2)}$$

The noise factor is

$$F = \frac{\overline{v_{onr}^2}}{\overline{v_{oni}^2}} = \frac{K^2\dfrac{r_i^2}{(r_i + R_S)^2}\left(\overline{v_n^2} + \overline{v_{nS}^2} + \overline{i_n^2}\,R_S^2\right)}{K^2\dfrac{r_i^2}{(r_i + R_S)^2}\,\overline{v_{nS}^2}} = \frac{\overline{v_n^2}}{4kTR_S} + 1 + \frac{\overline{i_n^2}\,R_S}{4kT} \qquad \textbf{(5.3)}$$

(i) This is simply a matter of putting numerical values into the real amplifier output noise expression,

$$\overline{v_{onr}^2} = K^2 \frac{r_i^2}{(r_i + R_S)^2} \left(\overline{v_n^2} + \overline{v_{nS}^2} + \overline{i_n^2} R_S^2 \right)$$

$$= 10^4 \times \frac{25}{49} \times (331 \times 10^{-18} + 144 \times 10^{-18} + 144 \times 10^{-18}) = 3.16 \times 10^{-12} \text{ V}^2 \text{ Hz}^{-1}$$

or $\boldsymbol{v_{onr}}$ **= 1.78 μV Hz$^{-1/2}$**

(ii) The signal to noise ratio at the output is

$$\frac{S_o}{N_o} = \frac{\text{output signal power}}{\text{output noise power}} = \frac{\text{mean squared output signal voltage}}{\text{mean squared output noise voltage}}$$

The mean squared output voltages are related to powers by the load resistance seen from the amplifier output according to $P_o = V_o^2 / R_L$. Since both voltages are measured at the same node of the circuit, R_L is the same for both and cancels to leave the ratio of mean squared voltages as shown. The mean squared signal voltage at the output is

$$S_o = (50 \times 10^{-6} \times \frac{5}{7} \times 100)^2 = 12.8 \times 10^{-6} \text{ V}^2$$

The mean squared noise voltage at the output over the bandwidth of interest can be found by taking the spectral density figure calculated in part **(i)**, 3.16×10^{-12} V^2 Hz^{-1}, and multiplying it by the system bandwidth of 10kHz to find the overall noise output. Thus,

$N_o = 3.16 \times 10^{-12} \times 10\text{kHz} = 31.6 \times 10^{-9}$ V^2

so S_o / N_o is **402 or 26 dB**.

(iii) The noise factor can be evaluated by putting suitable numbers into **(5.3)** or by using the definition

$$F = \frac{\text{noise output from the real amplifier}}{\text{noise output from an ideal version of the amplifier}}$$

$$= \frac{3.16 \times 10^{-12}}{4kTR_S \left(\frac{r_i}{r_i + R_S} \right)^2 100^2} = 1.87$$

Q6 (i)

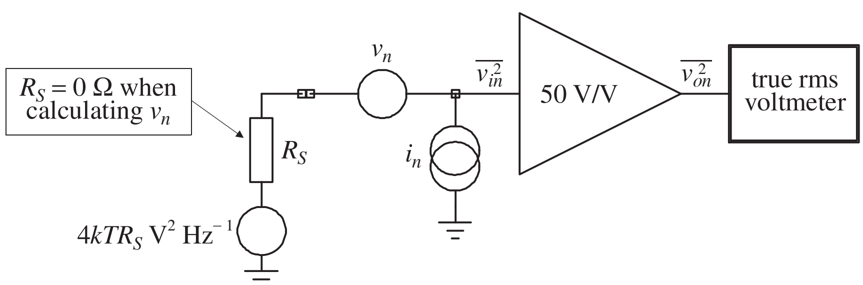


Figure 6

(ii) When the input is shorted to ground (ie $R_S = 0$),

$$\frac{(\text{meter reading})^2}{50^2 \text{x} \Delta f} = \overline{v_{in}^2} = \overline{v_n^2} \text{ or } v_n = \frac{\text{meter reading}}{50\text{x}(5\text{x}10^3)^{1/2}} = \mathbf{8.49\ nV\ Hz^{-1/2}}$$

When input comnnected to ground via a noisy 3 kΩ resistor,

$$\frac{(\text{meter reading})^2}{50^2 \text{x} \Delta f} = \overline{v_{in}^2} = \overline{v_n^2} + \overline{i_n^2} R_S^2 + 4kTR_S \text{ or } \boldsymbol{i_n} = \mathbf{2.95\ pA\ Hz^{-1/2}}$$

Q7 **(i)** The gains of matched amplifiers is specified assuming that the amplifier is fed from an impedance matched source and feeds an impedance matched load. If the two amplifiers have power gains A_{P1} and A_{P2}, the overall power gain is the product $A_{P1}A_{P2}$. If the amplifier gains are expressed in dB, a logarithmic form, the overall gain in dB is the sum of the two individual gains in dB.

The overall gain here is 25dB + 15 dB = 40dB, or in linear terms 316.2 x 31.62 = 10,000

(ii) The noise factor of each module can be found by remembering that

noise figure, $NF = 10 \log$ (noise factor, F) or $F = 10^{NF/10}$

The noise factors corresponding to noise figures of 4.5dB and 7dB are therefore 2.82 and 5.01 respectively.

(iii) The noise factor of a cascade of two matched amplifiers with individual gains and noise factors A_{P1}, F_1 and A_{P2}, F_2 respectively is:

$F_{overall} = F_1 + (F_2 - 1) / A_{P1} = 2.82 + 4.01 / 316.2 = 2.83$ in this case. The noise figure is therefore 10 log 2.83 = 4.52 dB.

Note how small an effect the second stage has on the overall system noise performance.

(iv) Treating the cascade as a single amplifier, $F_{overall} = 1 + N_{Aoverall} / A_{Poverall} \text{ x } N_i$. Using $F_{overall}$ = 2.83 (from part **(iii)**), $A_{Poverall}$ = 10,000 and $N_i = kT\Delta f$, $N_{Aoverall}$ = 75.8nW

(v) The definition of noise factor is the ratio of input to output signal to noise ratios. Thus if input signal to noise ratio and noise factor are known, it is easy to calculate output signal to noise ratio.

Available noise power at the input is $kT\Delta f = 1.38\text{x}10^{-23} \text{ x } 300 \text{ x } 10^9$ = 4.14pW so input signal to noise ratio is 10pW / 4.14pW = 2.42.

The noise factor, $F = (S_i/N_i) / (S_o/N_o) = 2.42 / (S_o/N_o) = 2.83$ (from part **(iii)**) which gives an output signal to noise ratio of 0.855 (or – 0.68dB).

(vi) The noise factor of the system can be written $F = 1 + T_E/T_A$ where T_E is the noise temperature of the amplifier and T_A is the ambient temperature. Thus $2.83 = 1 + T_E / 300$ or T_E = 549K