

# EEE225: Analogue and Digital Electronics

## Lecture VII

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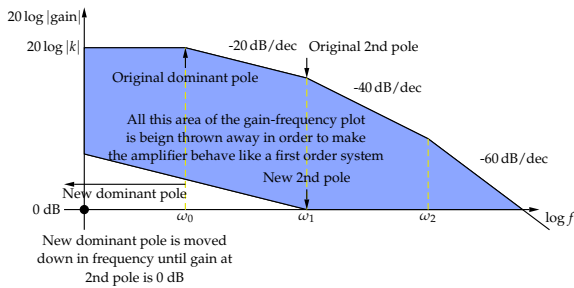
## This Lecture

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Frequency Dependence in Operational Amplifiers  
Engineering a First Order Open Loop Response



The dominant pole is moved down in frequency (sometimes called "slugging") until the 2nd pole frequency is at the unity gain point (0 dB). The blue area is open loop gain which is *lost*.

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First Order Opamp Model  
Model 'Derivation'

## First Order Opamp Model

In EEE118 we developed an equation for the opamp's operation

$$V_o = A_v (v^+ - v^-) \quad (1)$$

We now have an expression for  $A_v$  as well

$$A_v = \frac{A_0}{1 + j \frac{\omega}{\omega_0}} \text{ where } \tau = \frac{1}{\omega_0} = \frac{1}{2\pi f_0} \text{ or } f_0 = \frac{\omega_0}{2\pi} = \frac{1}{2\pi\tau} \quad (2)$$

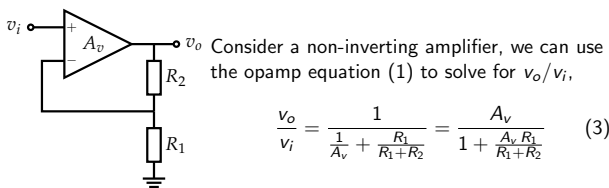
where  $A_0$  is the DC open loop gain and  $\omega_0$  is the angular corner frequency of the first order system ( $\text{rads}^{-1}$ ).

- $A_0$  is usually between  $10^4$  and  $10^7$
- $\omega_0$  is typically  $2\pi \cdot 10$  Hz.

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First Order Opamp Model  
Model 'Derivation'



Consider a non-inverting amplifier, we can use the opamp equation (1) to solve for  $v_o/v_i$ ,

$$\frac{v_o}{v_i} = \frac{1}{\frac{1}{A_v} + \frac{R_1}{R_1 + R_2}} = \frac{A_v}{1 + \frac{A_v R_1}{R_1 + R_2}} \quad (3)$$

Use an equation for  $A_v$  (2) and seek a standard form,

$$\frac{v_o}{v_i} = \frac{\frac{A_0}{1 + \frac{A_0 R_1}{R_1 + R_2}}}{1 + j \frac{\omega}{\omega_0 \left(1 + \frac{A_0 R_1}{R_1 + R_2}\right)}} \equiv \frac{k_1}{1 + j \omega k_2} \equiv \frac{k_1}{1 + j \frac{\omega}{\omega_2}} \quad (4)$$

$$\text{where } k_1 = \frac{A_0}{1 + \frac{A_0 R_1}{R_1 + R_2}} = \text{new d.c. gain} \approx \frac{R_1 + R_2}{R_1} \text{ for } A_0 \gg 1 \quad (5)$$

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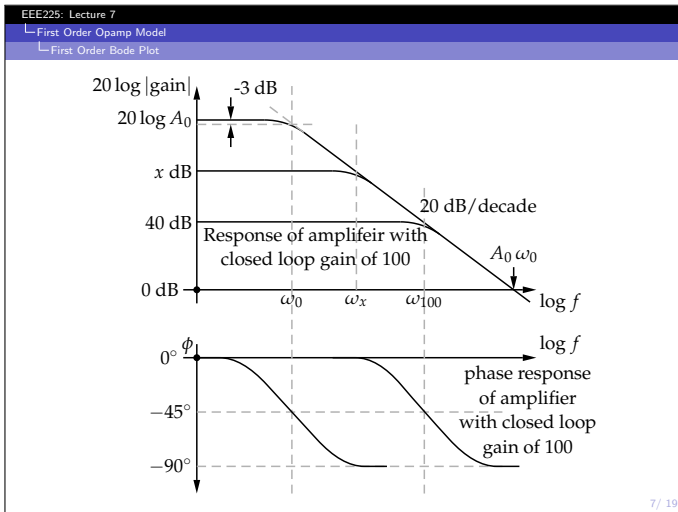
First Order Opamp Model  
Model 'Derivation'

The product  $\omega_2 k_1$  is called the gain-bandwidth product of the amplifier.

$$\omega_2 k_1 = \omega_0 \left(1 + \frac{A_0 R_1}{R_1 + R_2}\right) \cdot \frac{A_0}{1 + \frac{A_0 R_1}{R_1 + R_2}} = A_0 \omega_0 \quad (6)$$

So  $A_0 \omega_0$  is constant for a particular opamp. This is a very important result. It means that the product of the d.c. gain and the -3dB bandwidth for a single pole op-amp is independent of the feedback resistor values (and hence closed loop gain) and therefore is a property of the op-amp itself. We can use this idea to make rapid estimates of bandwidth for a given gain or *vice versa*. The consequence of this constant gain bandwidth product can be visualised in a graph showing amplifiers having several different gains.

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 First Order Opamp Model  
 Key points about the Model

**Key points about the Model**

- The only information given by manufacturers is the gain bandwidth product or the unity gain frequency. Anything else can be calculated by remembering the opamp is assumed to be first order.
- All the roll-offs follow the open loop curve
- Each of the three responses shown exhibits first order behaviour
- For the open loop response, the product of the DC gain and the -3 dB bandwidth is equal to the product of gain and -3 dB frequency for both closed loop gains shown in the last slide.
- For the closed loop gains shown, the product of DC gain and -3 dB frequency is constant and is equal to the unity gain bandwidth of the open loop response.

**DC gain × -3 dB bandwidth = open loop unity gain frequency = constant**

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 First Order Opamp Model  
 Key points about the Model

**Yet more... Key points about the Model**

- The first order approximation only applies to non-inverting amplifiers which the manufacture describes as **unity gain compensated**.
- Real opamps are generally not actually first order. There is often one or more poles associated with each stage (3 stages = 3 poles). If, over the range of frequencies where  $A_v > 1$ , only one pole dominates then the response will be first order.
- The manufacture often chooses to make the VAS stage pole the dominant one by adding a capacitance between the collector and base of the VAS transistor often between 10 pF and 30 pF.
- The effect of this capacitance is magnified from the VAS's perspective by **Miller multiplication**.
- Miller multiplication can be described by the **Miller Transform**.

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 First Order Opamp Model  
 Easy example of GBP in a Calculation

**'Easy' example**

You require a gain of 100 from a TL081 (the opamp in the amplifier lab). It has a gain bandwidth product of 3 MHz. What will be the rise time of the circuit be in response to a 10 mV step input?

$$f_{-3dB} = \frac{3 \text{ MHz}}{100} = 30 \text{ kHz} \quad (7)$$

$$\text{time constant, } \tau = \frac{1}{\omega_0} = \frac{1}{2\pi f_0} \quad (8)$$

$$= \frac{1}{2\pi \cdot 30 \text{ kHz}} \quad (9)$$

$$\text{rise time} = 2.2 \tau \quad (10)$$

$$\text{rise time} = 2.2 \cdot 4 = 11.17 \mu\text{s} \quad (11)$$

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 First Order Opamp Model  
 Exam style example of GBP in a Calculation

**Exam style example - part 1**

A particular amplifier with a dc gain of 100 V/V is observed by experiment to behave like a first order system. Measurement shows that the magnitude of amplifier gain has dropped to -6 dB at a frequency of 120 kHz. Calculate the -3 dB frequency.

The amplifier is first order so it will obey

$$\frac{v_o}{v_i} = k \frac{1}{1 + j \frac{\omega}{\omega_0}} \text{ where } k = 100. \quad (12)$$

$$\left| \frac{100}{1 + j \frac{120 \times 10^3}{f_0}} \right| = 50 \text{ or } \frac{1}{1 + \left( \frac{120 \times 10^3}{f_0} \right)^2} = \left( \frac{1}{2} \right)^2 \quad (13)$$

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 First Order Opamp Model  
 Exam style example of GBP in a Calculation

$$\sqrt{4 - 1} = \frac{120 \times 10^3}{f_0} = 1.73, f_0 = \frac{120 \times 10^3}{1.73} = 69.4 \text{ kHz} \quad (14)$$

$$\text{GBP} = 100 \cdot 69.4 \text{ kHz} = 6.94 \text{ MHz} \quad (15)$$

**Exam style example - part 2**

A different amplifier also having a dc. gain of 100 V/V has a GBP of 100 kHz. Evaluate the |gain| and phase shift of this amplifier at 75 kHz.

We can use the GBP to get the -3 dB frequency.  $f_0 = 100 \text{ kHz} / 100 \therefore f_0 = 1 \text{ kHz}$ .

$$\frac{v_o}{v_i} = k \frac{1}{1 + j \frac{\omega}{\omega_0}} = \frac{100}{1 + j \frac{f}{1 \times 10^3}} \quad (16)$$

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$$\text{At } 75 \text{ kHz, } |A| = \left| \frac{v_o}{v_i} \right| = \frac{100}{\left[ 1 + \left( \frac{75}{1} \right)^2 \right]^{\frac{1}{2}}} \quad (17)$$

$$\left| \frac{v_o}{v_i} \right| = 1.33\dot{3} \text{ V/V or } 2.498 \text{ dBV} \quad (18)$$

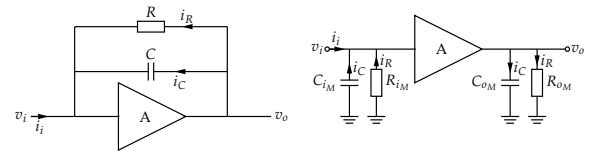
$$\text{The phase shift, } \theta = \angle \left( \frac{v_o}{v_i} \right) = -\tan^{-1} \left( \frac{f}{f_0} \right) \quad (19)$$

$$= -\tan^{-1} \left( \frac{75}{1} \right) = -89.236^\circ \quad (20)$$

Where does the minus sign come from in (19)? "Proper" derivation of argument of complex number needed... Biscuits for correct answers!

### Miller Transform

For the two generic amplifier circuits with RC feedback (below), the Miller transform aims to find the effective value of C and R from the point of view of the amplifier's input source and the amplifier's load. The effective values of C and R become  $C_{OM}$  and  $R_{OM}$  for the output and  $R_{iM}$  and  $C_{iM}$  for the input. In other words, find a value of  $C_{iM}$  and  $R_{iM}$  which makes  $\frac{v_i}{i_i}$  the same for both circuits. Similar arguments with  $R_{OM}$  and  $C_{OM}$  for  $\frac{v_o}{i_o}$ .



In the feedback amplifier,

$$i_R = \frac{v_o - v_i}{R} \text{ and } i_C = (v_o - v_i) j\omega C \quad (21)$$

In the Miller transformed amplifier, for  $i_R$ ,

$$i_R = \frac{0 - v_o}{R} = -\frac{v_i}{R_{iM}} \text{ or } \frac{A v_i - v_i}{R} = -\frac{v_i}{R_{iM}} \quad (22)$$

$$\text{so } R_{iM} = \frac{R}{1 - A} \quad (23)$$

and for  $i_C$

$$i_C = (v_o - v_i) j\omega C = -v_i j\omega C = -v_i j\omega C_{iM} \quad (24)$$

$$\text{so } C_{iM} = C(1 - A) \quad (25)$$

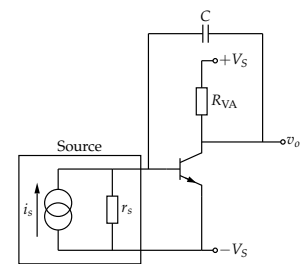
Using a similar analysis, the value of R to ground at the output is  $R_{OM} = (A / (A - 1)) \cdot R$  and the value of C to ground is  $C_{OM} = ((A - 1) / A) \cdot C$ .

### Miller Multiplication and the VAS

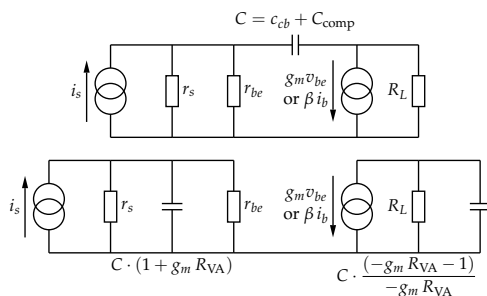
From the point of view of the signal source, the feedback impedance is  $(1 - A)$  times lower than the component face value.

In the amplifier on the right, the feedback capacitor's value is effectively multiplied by  $(1 - A)$ . This is the Miller multiplication. The gain of this amplifier is approximately  $-g_m R_{VA}$ , so the apparent value of the capacitor is increased by  $(1 - g_m R_{VA})$ . For  $500 \mu\text{A}$  quiescent current and  $R_{VA}$  a quite conservative  $50 \text{ k}\Omega$ , if  $C = 33 \text{ pF}$  its apparent value

will appear as  $326 \text{ nF}$ . We use the Miller effect to our advantage when lowering the pole frequency of the VAS...



- $(1 - g_m R_{VA})$  is usually large, especially if the VAS is a Darlington (could be  $> 50000$ ).
- If the VAS is a Darlington then  $r_{be}$  will be very large, this gives rise to a very long time constant,  $\tau$ , as seen from the output of the differential stage.



### Review

- Reminded ourselves how to read a Bode plot.
- Discussed how the higher order opamp is made to "look" first order.
- Developed an expression for the open loop gain as a first order low pass system.
- Introduced the idea of **gain bandwidth product**.
- Did a simple example GBP / rise time calculation.
- Noted some key points about the first order model.
- Developed the idea that the VAS's  $C_{cb}$  is increased in order to **compensate** the amplifier.
- Introduced the Miller Transform, considered the advantage brought by the Miller Effect for dominant pole positioning.

