Frequency Dependent Aspects of Op-amps

Frequency dependent feedback circuits

The arguments that lead to expressions describing the circuit gain of inverting and non-inverting amplifier circuits with resistive feedback apply also to the more general case of a feedback network made of impedances.

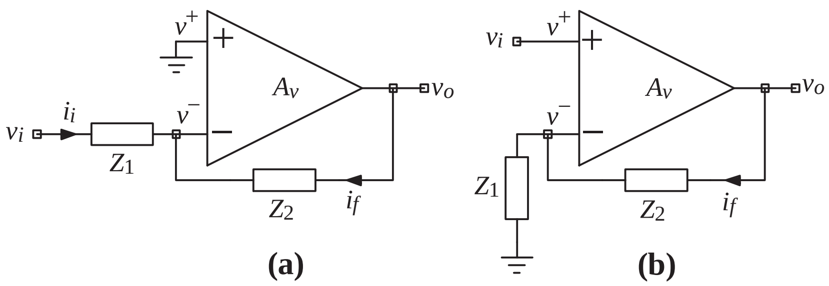


Figure 1
Op-amp circuits with complex impedance in the feedback circuit

For the inverting circuit of figure 1a, summing currents at the v^- node (assuming that the op-amp draws no input current) gives

$$i_i + i_f = 0 \text{ or } \frac{v_i - v^-}{Z_1} = \frac{v_o - v^-}{Z_2}$$

Since $A_v \Rightarrow \infty$, $v^- \approx v^+$ and since $v^+ = 0$, the circuit gain reduces to $\frac{v_o}{v_i} = -\frac{Z_2}{Z_1}$. **(1)**

For the non-inverting circuit of figure 1b, v^- is the potential division of v_o by Z_2 and Z_1 (again assuming that the op-amp draws no input current) and so

$$v^- = v_o \frac{Z_1}{Z_1 + Z_2}$$

Since $A_v \Rightarrow \infty$, $v^- \approx v^+$ and since $v^+ = v_i$, the circuit gain reduces to $\frac{v_o}{v_i} = \frac{Z_1 + Z_2}{Z_1}$. **(2)**

Although these results assume a perfect amplifier, they are valid for real amplifiers providing that the amplifier gain at the frequencies of interest is sufficiently high. In other words, as with resistive feedback, the circuit gain must be controlled by the feedback elements and not by the amplifier itself (to any significant extent) if the circuit is to be useful.

Notice that no restriction has been placed on the *Z*s - that simply means that the analytical approach to analysis does not put restrictions on the nature of *Z*. It does not imply that one can use any old *Z*; frequency dependent *Z* has the potential to cause instability. The rest of this section looks at a number of standard frequency dependent circuits.

The Integrator

The integrator was the workhorse circuit of analogue computers - it was central to the process of solving differential equations. These days it is its frequency dependent behaviour that is of interest. Integrators are used in filtering circuits and in instrumentation circuits. The circuit diagram of an integrator circuit is shown in figure 2. There are two ways of deriving a relationship between input and output of this circuit; one is a frequency domain analysis and the other is a time domain analysis. The current and voltage variables shown in figure 2 are appropriate for the frequency domain analysis. For the time domain analysis, upper case versions of the symbols are used - eg, i_i for the frequency domain becomes I_I for the time domain.

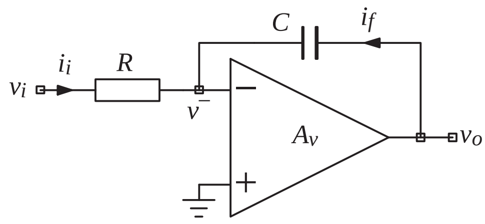


Figure 2

The op-amp integrator

Frequency domain analysis of an integrator

The circuit of figure 2 has the same shape as that of figure 1a. Z_2 is the impedance of C and Z_1 is simply R. The same same approximations regarding A_v that were used to derive equation **(1)** must be valid if the integrator is to be useful so the gain of the circuit is as described by equation **(1)**,

$$\frac{v_o}{v_i} = -\frac{\left(\frac{1}{j\omega C}\right)}{R} = -\frac{1}{j\omega CR}$$

It is quite common for the j and ω to be kept together and given the symbol s. Thus

$$\frac{v_o}{v_i} = -\frac{1}{j\omega CR} = -\frac{1}{sCR} \qquad \textbf{(3)}$$

The frequency response of the integrator in both magnitude and phase is shown in figure 3. The horizontal axis is a logarithmic frequency scale normalised to the frequency $\omega = 1/CR$. The ideal responses that describe equation **(3)** are the black lines. Since equation **(3)** suggests that

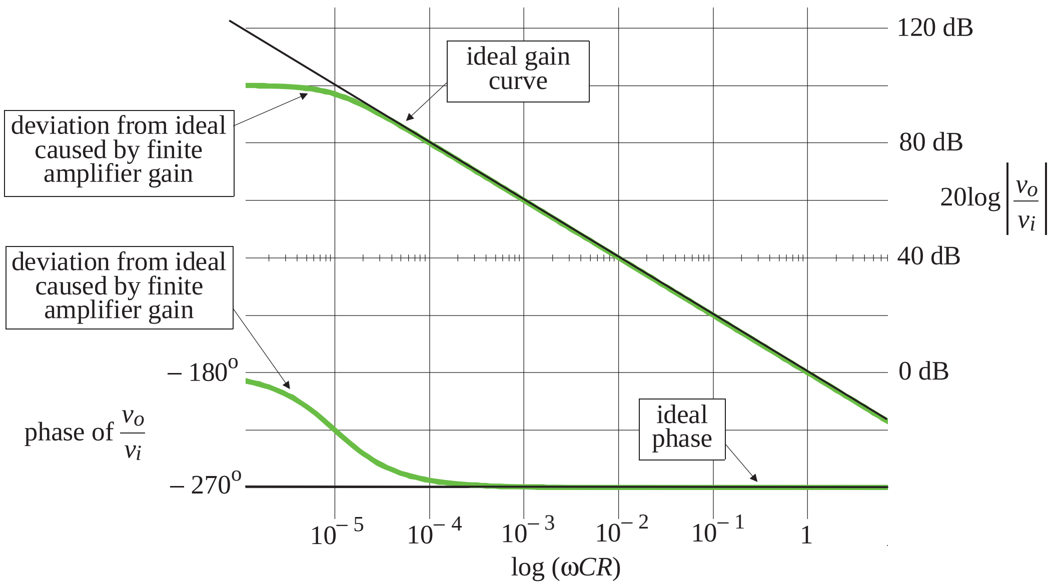


Figure 3

Amplitude and phase response of an ideal and a real integrator as ω approaches zero, gain approaches infinity there must be some frequency at which the finite gain of real amplifiers affects performance. This is shown by the slightly lighter coloured curves that deviate from the ideal straight lines in the region of 10^{-4} to 10^{-6} on the normalised frequency scale. The integrator must be used in a frequency range where the magnitude and phase errors due to finite gain effects are negligible - for example, 10^{-3} or greater in the case of figure 3.

Time domain analysis of an integrator

For the time domain treatment of the integrator, current is again summed at the inverting input,

$$I_I + I_F = \frac{V_I - V^-}{R} + C\,\frac{d\,(V_O - V^-)}{dt} = 0 \qquad \textbf{(4)}$$

where $V_O - V^-$ is the voltage across the capacitor. Assuming (as for the frequency domain case) that $V^- \approx 0$, equation **(4)** simplifies to

$$\frac{V_I}{R\,C} = -\frac{dV_O}{dt} \qquad \textbf{(5)}$$

We want to find V_O in terms of V_I. Integrating both sides of equation **(5)** gives

$$V_O = -\frac{1}{C\,R}\int V_I\,dt + A \qquad \textbf{(6)}$$

where A is a constant of integration that represents the voltage across the capacitor due to its initial charge - the charge in the capacitor at the start of the integration interval.

Problems with integrators

The biggest problem with the integrator circuit arises because of its lack of dc feedback. Any small residual dc input, which may arise in the signal source or may exist at the op-amp input as an equivalent offset generator, will be integrated until the output magnitude is limited by the power supply voltage; in other words, the output saturates at one or other of the power supply voltages in the absence of dc feedback.

In some applications the integrator forms part of a larger analogue system such that the rest of the system provides the dc feedback that the integrator needs. An example of this is using the integrator to emulate the behaviour of a first order *RC* circuit such as that of figure 4. V_O and I are given by

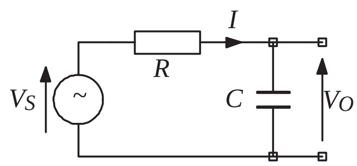


Figure 4

A simple RC circuit

$$V_O = \frac{1}{C}\int I\,dt \quad \text{and} \quad I = \frac{V_S - V_O}{R}$$

and these two relationships can be combined to give

$$V_O = \frac{1}{CR}\int (V_S - V_O)\,dt = -\frac{1}{CR}\int (V_O - V_S)\,dt \qquad \textbf{(7)}$$

Equation **(7)** can be "solved" by the circuit of figure 5. Start by assuming that V_O exists. If it exists, it must be the result of integrating the difference between V_O and V_S. The difference is performed symbolically as shown. The summing amplifier in this case could be an op-amp subtracting circuit with a gain of 1. Dc feedback is built in to the loop from the integrator output to the summing amplifier input. This approach to realising differential equation solutions formed the basis of analogue computers

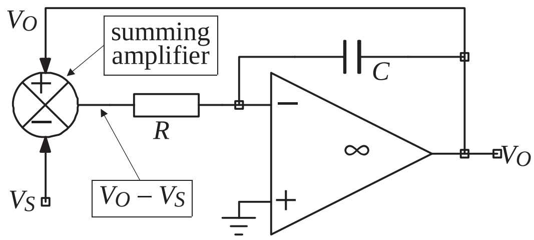


Figure 5

An integrator based system for solving a first order differential equation

Some dc feedback can be added to the basic integrator circuit by adding a resistor in parallel with C as shown in figure 6. This is an effective strategy in terms of stabilising the dc conditions in the circuit but it has an undesirable effect on the integration function. The dc gain of the integrator is effectively reduced from the open loop dc gain of the op-amp, $-A_0$, to a lower value, $-R_F/R$. The effect of this gain lowering is to move the deviations to gain and phase caused by finite amplifier gain (shown in figure 3) upwards in frequency thereby reducing the range of frequencies over which the integrator is useful.

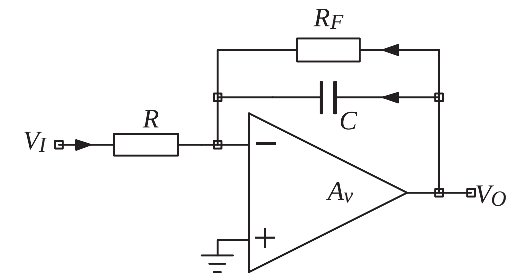


Figure 6

An integrator circuit with a feedback resistor to provide dc feedback

The Differentiator

If integration is possible with the help of an op-amp, it is reasonable to assume that differentiation is also possible. Consider the circuit of figure 7. The analytical process is the same as for the integrator except that the two key components have been interchanged. Using the same approximations and assumptions as were used for the integrator, the frequency domain approach leads to

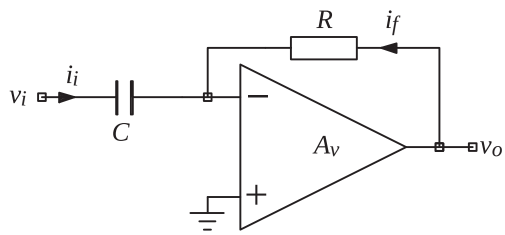


Figure 7

An op-amp differentiator circuit

$$\frac{v_o}{v_i} = -j\omega CR = -sCR \qquad \textbf{(8)}$$

and for the time domain

$$V_O = CR\frac{dV_i}{dt} \qquad \textbf{(9)}$$

Although the differentiator looks attractive on the basis of this simple analysis, in fact the feedback components interact with the internal frequency response of the op-amp (which will be dealt with later) to form an underdamped second order (resonant) system. This interaction makes the differentiator virtually useless over a very wide range of operating frequency. Figure 8 shows the transient response of a typical op-amp differentiator to a triangular input signal. The output should be a rectangular waveform but notice how each change of slope is followed on the output trace by a lightly damped sinusoid. The circuit is stable but seriously underdamped.

It is possible to control the circuit damping by adding a resistor in series with C or a capacitor in parallel with R but in controlling the damping the circuit becomes ineffective as a differentiator.

Differentiators also have a reputation for being noisy circuits - a slightly unfair reputation. White noise, the most common form of noise, has a constant power per unit bandwidth so any system that has a gain that increases with frequency will be subject to the same "noisy" problem.

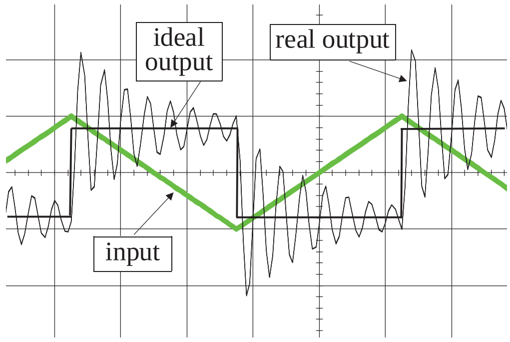


Figure 8

Ideal and real integrator output waveforms

Pole-zero (or lead-lag or lead-lag) circuits

These circuits are commonly used to manipulate the frequency or the phase response of electronic systems. The basic amplifier circuits of figure 1 are repeated here as figure 9 for convenience. The impedances Z_1 and Z_2 may be real or complex but there will only be one capacitor in the circuit and one of Z_1 and Z_2 will be real (ie, purely resistive). A couple of examples are given below.

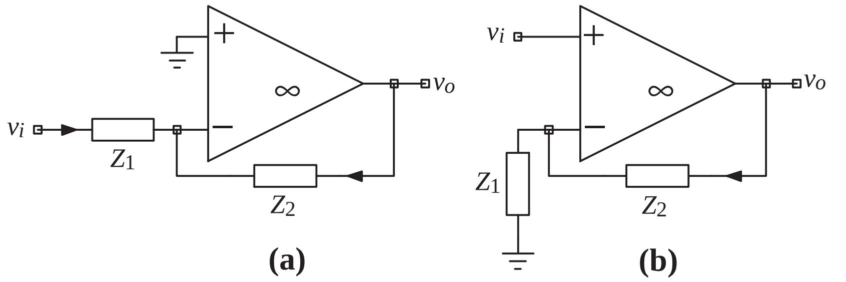


Figure 9
Op-amp circuits with complex impedance in the feedback circuit

Example 1

The circuit of figure 10 shows a non-inverting amplifier with frequency dependent feedback. Notice that the gain of the amplifier is assumed to approach infinity so the circuit behaviour is controlled by the feedback components. The gain of the non-inverting amplifier is given by equation **(2)** as

$$\frac{v_o}{v_i} = \frac{Z_1 + Z_2}{Z_1}$$

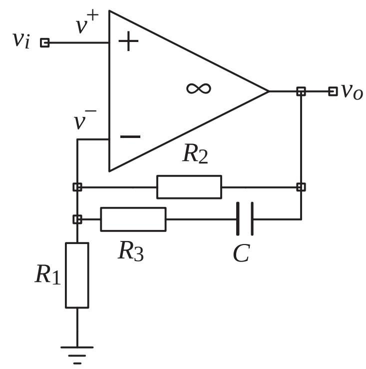


Figure 10

A non-inverting pole-zero circuit

In this circuit $Z_1 = R_1$ and $Z_2 = R_2 \,//\, (R_3 + Z_C)$ so

$$\frac{v_o}{v_i} = \frac{R_1 + \dfrac{R_2\left(R_3 + \dfrac{1}{j\omega C}\right)}{R_2 + R_3 + \dfrac{1}{j\omega C}}}{R_1} = \frac{R_1 + \dfrac{R_2(j\omega C R_3 + 1)}{j\omega C(R_2 + R_3) + 1}}{R_1} \qquad \textbf{(10)}$$

The aim is to reduce the gain expression to a ratio of two polynomials in $j\omega$. It is important to keep j and ω together in this process. Simplifying further and collecting real and imaginary terms gives

$$\frac{v_o}{v_i} = \frac{R_1 + R_2 + j\omega C(R_1R_2 + R_1R_3 + R_2R_3)}{R_1(j\omega C(R_2 + R_3) + 1)} = \frac{R_1 + R_2}{R_1} \; \frac{1 + j\omega C\left(\dfrac{R_1R_2 + R_1R_3 + R_2R_3}{R_1 + R_3}\right)}{1 + j\omega C(R_2 + R_3)} \quad \textbf{(11)}$$

In equation **(11)**, a complex number of the form $a + jb$ has been modified to $a(1 + jb/a)$. This apparently pointless manipulation makes the frequency dependent function easier to interpret. If the time $C\dfrac{R_1R_2 + R_1R_3 + R_2R_3}{R_1 + R_2}$ is written as a frequency domain constant $\dfrac{1}{\omega_1}$ and the time $C(R_2 + R_3)$ is written as a frequency domain constant $\dfrac{1}{\omega_0}$, equation **(11)** can be rewritten as

$$\frac{v_o}{v_i} = k\,\frac{1 + j\dfrac{\omega}{\omega_1}}{1 + j\dfrac{\omega}{\omega_0}} = k\,\frac{1 + j\dfrac{f}{f_1}}{1 + j\dfrac{f}{f_0}} \quad \textbf{(12)}$$

Equation **(12)** is in a first order "standard form". To interpret this transfer function we need to look at the basic behaviour of first order standard forms.

First order standard forms

First order transfer functions fall into one of the three standard forms:

low pass $$\frac{v_o}{v_i} = k\,\frac{1}{1 + j\dfrac{\omega}{\omega_0}} \quad \textbf{(13)}$$

high pass $$\frac{v_o}{v_i} = k\,\frac{j\dfrac{\omega}{\omega_0}}{1 + j\dfrac{\omega}{\omega_0}} \quad \textbf{(14)}$$

pole-zero $$\frac{v_o}{v_i} = k\,\frac{1 + j\dfrac{\omega}{\omega_1}}{1 + j\dfrac{\omega}{\omega_0}} \quad \textbf{(15)}$$

Equation **(15)** is the sum of a low pass, equation **(13)**, and a high pass, equation **(14)**, response with different frequency independent gains. The frequency responses for equations **(13)**, **(14)** and **(15)** are shown in figures 11, 13 and 15. The corresponding transient responses are shown in figures 12, 14 and 15.

In first order low pass and high pass circuits, the angular frequency ω_0, which is the reciprocal of time constant, is called the corner frequency - a brief inspection of figures **11** and **13** will reveal why. The pass band gain is given by 20log$k$ dB. Notice how good the straight line approximations (also known as Bode approximations) are to both the amplitude and phase responses. **Once the transfer function has been reduced to a standard form, ω_0, k and the response type completely specify the response shape.**

Low Pass

Frequency response :-

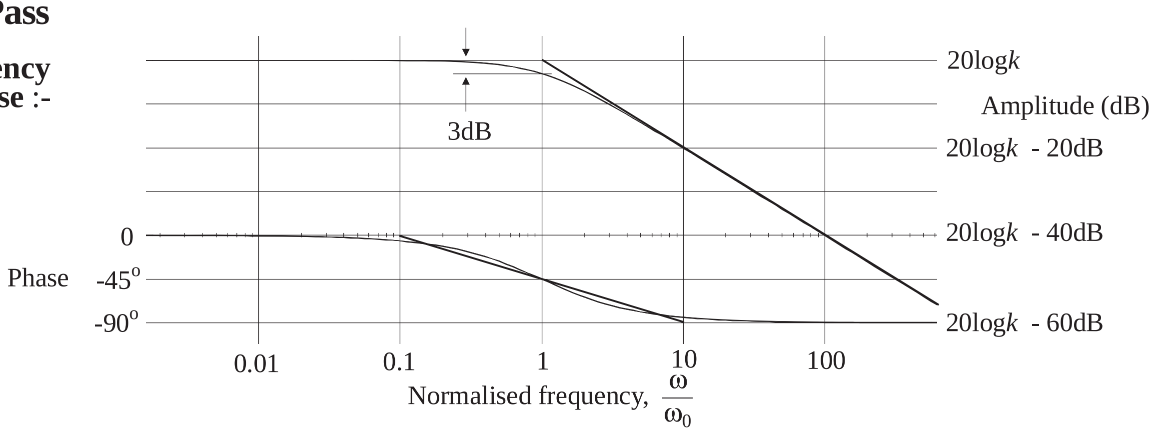


Figure 11

Frequency response of a first order low pass circuit. The corner frequency and gain, k, completely specify the response. The curves of gain magnitude and phase are real responses and the straight line approximations to them are called Bode approximations.

Transient response :-

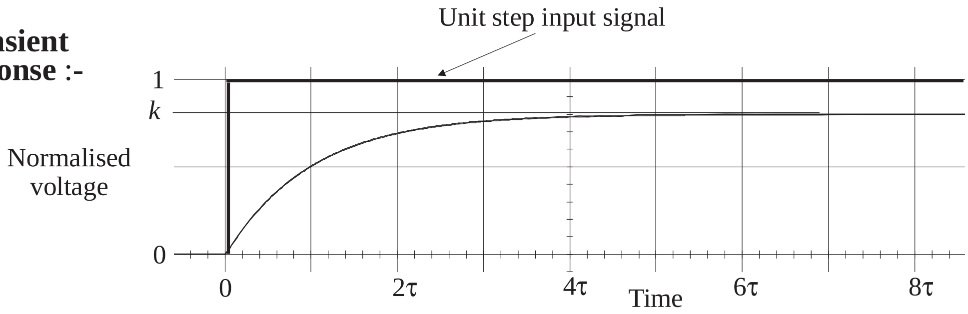


Figure 12

Step response of a first order low pass circuit. The two pieces of information that completely specify the response are step amplitude and circuit time constant.

High Pass

Frequency response :-

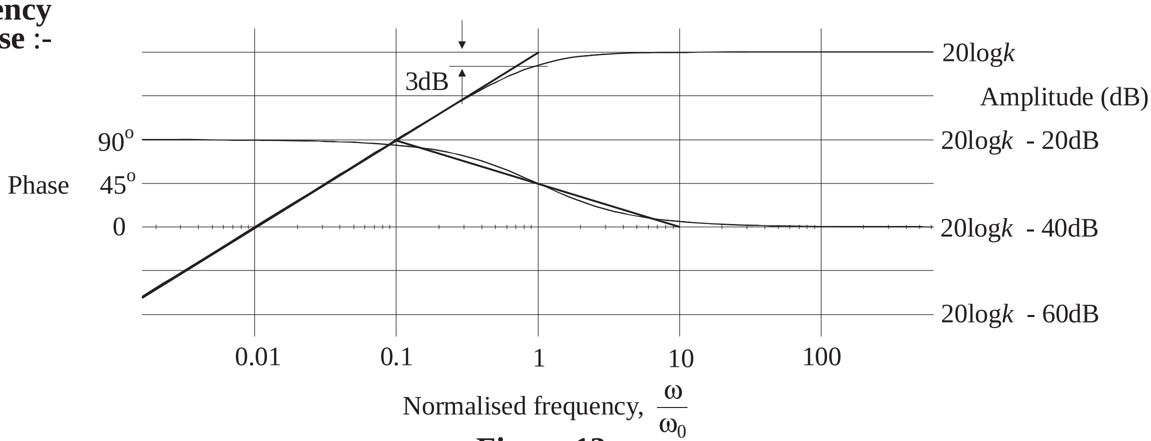


Figure 13

Frequency response of a first order high pass circuit. Again the high-pass response is completely specified by knowledge of corner frequency and k. The Bode approximations are shown on the graphs.

Transient response :-

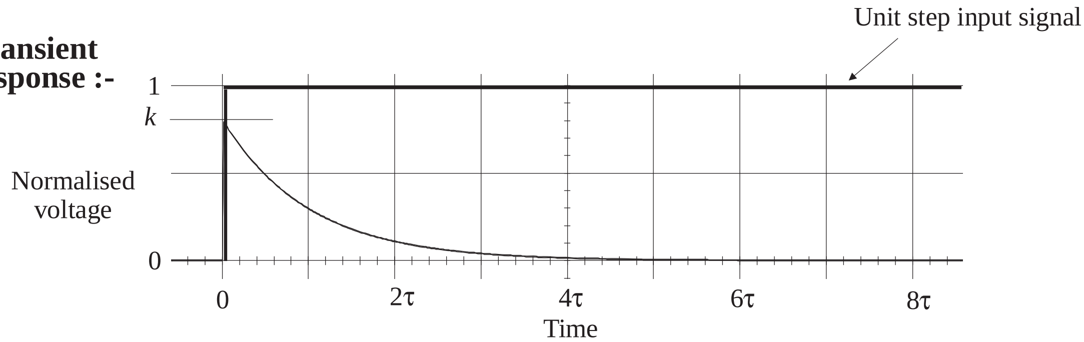


Figure 14

Step response of a high pass circuit. The response is completely specified by k and time constant or corner frequency.

The transient responses are easily drawn once the response type (high pass or low pass) has been identified. A circuit described by a low pass transfer function will always have a unit step (transient) response of the form $v(t) = k\,(1 - \exp(t/\tau))$ whereas one described by a high pass transfer function will always have a unit step response of the form $v(t) = k \exp(t/\tau)$. In both cases $\tau = 1/\omega_0$. In the low pass case the frequency independent gain k determines the aiming level of the exponential rise while in the high pass case k determines the initial height of the exponential waveshape. If the amplitude of the input step is V_S, the initial height and aiming levels become kV_S instead of k.

Pole - Zero

The pole-zero standard form is a linear sum of high pass and low pass forms, each being multiplied by a different frequency independent gain. Equation **(15)** can be expanded as follows:

$$\frac{v_o}{v_i} = k\,\frac{1 + j\frac{\omega}{\omega_1}}{1 + j\frac{\omega}{\omega_0}} = k\,\frac{1}{1 + j\frac{\omega}{\omega_0}} + k\,\frac{j\frac{\omega}{\omega_1}}{1 + j\frac{\omega}{\omega_0}} = k\,\frac{1}{1 + j\frac{\omega}{\omega_0}} + \frac{k\,\omega_0}{\omega_1}\,\frac{j\frac{\omega}{\omega_0}}{1 + j\frac{\omega}{\omega_0}} \qquad \textbf{(16)}$$

The form of equation **(15)** is most useful from the point of view of frequency responses but the final form of equation **(16)** is most useful from the point of view of transient responses. The gain multiplying the low pass part of equation **(16)** - the low frequency gain - can in general be larger than or smaller than the gain multiplying the high pass part of equation **(16)** - the high frequency gain. In some cases one is always higher than the other.

The pole-zero response of figure 15 is drawn for a circuit where the low frequency gain is lower than the high frequency gain or, in other words, in terms of equation **(15)**, $\omega_1 < \omega_0$. The high frequency gain, k_H, is $k_L\omega_0/\omega_1$ as in the final form of equation **(16)**. Notice the parts ω_0, ω_1, k_L and k_H play in the response shapes - once again they provide the coordinates needed for a straight line approximation to the real curve and define the initial and aiming levels of the transient response.

The phase response appears here as a positive going hump. It gets close to 90^o only if ω_0 and ω_1 are widely spaced (say two orders of magnitude or more). It is not easy to make an accurate sketch of the phase response for pole-zero circuits without evaluating some points on the phase-frequency graph. This shape of phase response is useful as compensation to ensure stability in feedback systems and is commonly used in such applications.

The step response moves exponentially from its initial value to its aiming level with a time constant of $1/\omega_0$. **Notice that it is always the denominator of the transfer function that gives the circuit time constant.**

If $\omega_0 < \omega_1$ the high frequency gain is lower than the low frequency gain, so the gain falls between ω_0 and ω_1. The phase is a negative going hump - essentially an upside down version of the shape shown. The transient response becomes a rising exponential starting with an initial step of k_H (for a unit step input) and aiming for a level of k_L with a time constant of $1/\omega_0$.

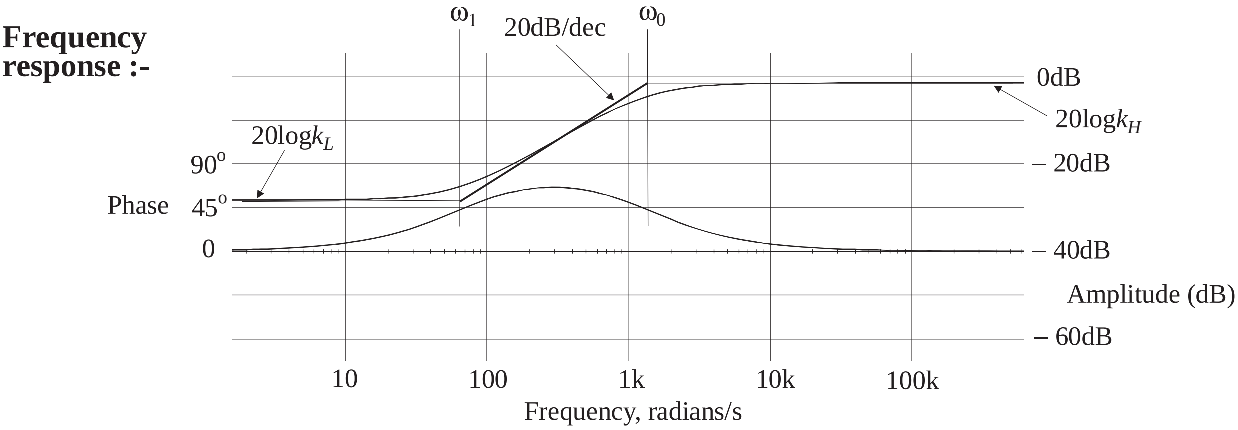


Figure 15

A pole-zero response for a circuit where low frequency gain is lower than high frequency gain. Notice that the gain approaches steady values at low frequencies and at high frequencies

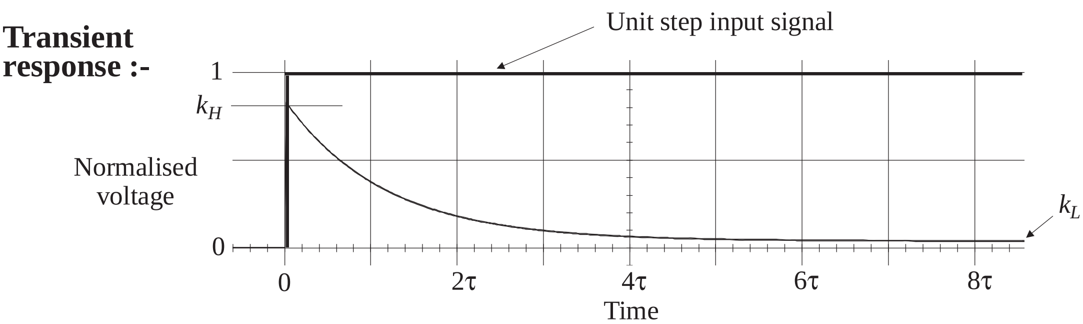


Figure 16

The step response of a pole-zero circuit. Notice that the gain of the low pass part of the circuit determines the aiming level whilst the initial step height is controlled by the high-pass gain. All amplitude values are proportional to the step height.

Returning to example 1 of figure 10 . . . .

The response can be identified as a pole-zero response by comparing the transfer function of equation **(12)** with the three standard forms of equations **(13)**, **(14)** and **(15)**. In the circuit of figure 10 the high frequency (h.f.) gain is lower than the low frequency (l.f.) gain. This fact can be deduced as follows:

At high frequencies, the reactance of C is much less than the resistances in the circuit so C can be regarded as a short circuit. This leads to a feedback path with an effective resistance of $R_2//R_3$. The h.f. gain is thus $\frac{v_o}{v_i} = \frac{R_1 + (R_2//R_3)}{R_1} = 1 + \frac{R_2//R_3}{R_1}$. The l.f. gain can be found by letting the capacitive reactance become much larger than the surrounding resistors. The feedback path now has an effective resistance of R_2 and the gain becomes $\frac{v_o}{v_i} = \frac{R_1 + R_2}{R_1} = 1 + \frac{R_2}{R_1}$. Since $R_2//R_3$ must be smaller than R_2, the h.f. gain must be smaller than the l.f. gain. It is always worth performing this quick estimate of high and low frequency gains as a check of your analysis.

It is also possible to find the h.f. and l.f. gain from equation **(12)** by letting ω approach zero for l.f. and infinity for h.f.. The modulus of equation **(12)** is

$$\left|\frac{v_o}{v_i}\right| = k \left[\frac{1+\frac{\omega^2}{\omega_1^2}}{1+\frac{\omega^2}{\omega_0^2}}\right]^{\frac{1}{2}} \qquad \textbf{(17)}$$

If $\omega \ll \omega_1$ and $\omega \ll \omega_0$ then the circuit gain approaches k and this is the l.f. gain. If $\omega \gg \omega_1$ and $\omega \gg \omega_0$ then the circuit gain approaches $k\ \omega_0/\omega_1$ and this is the h.f. gain.

Example 2

The circuit of figure 17 is an inverting amplifier. Making the assumption that A_V approaches infinity, v_o/v_i is given by equation **(1)**,

$$\frac{v_o}{v_i} = -\frac{Z_2}{Z_1}$$

In this circuit, $Z_1 = R_1 + (R_3//C)$ and $Z_2 = R_2$. Thus

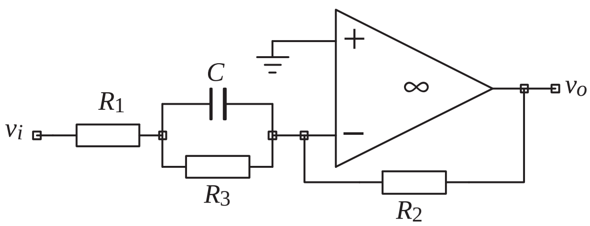


Figure 17
A inverting pole-zero circuit

$$Z_1 = R_1 + \frac{\frac{R_3}{j\omega C}}{R_3 + \frac{1}{j\omega C}} = R_1 + \frac{R_3}{1 + j\omega C R_3}$$

and circuit gain is given by

$$\frac{v_o}{v_i} = -\frac{R_2}{R_1 + \frac{R_3}{1+j\omega C R_3}} = -\frac{R_2\,(1+j\omega C R_3)}{R_1 + j\omega C R_1 R_3 + R_3}$$

$$= -\frac{R_2\,(1+j\omega C R_3)}{R_1 + j\omega C R_1 R_3 + R_3} = -\frac{R_2\,(1+j\omega C R_3)}{(R_1+R_3)\left(1+j\omega C\frac{R_1 R_3}{R_1+R_3}\right)} = -k\,\frac{1+j\frac{\omega}{\omega_1}}{1+j\frac{\omega}{\omega_0}} \qquad \textbf{(18)}$$

where $k = \frac{R_2}{R_1+R_3}$, $\omega_1 = \frac{1}{CR_3}$ and $\omega_0 = \frac{R_1+R_3}{C\,R_1 R_3}$

Once again the transfer function can be expressed in the standard form given by equation **(15)** which allows figure 17 to be identified as a pole-zero circuit. By inspection the l.f. gain (when C looks like an open circuit) is $-R_2/(R_1+R_3)$ and the h.f. gain (when C looks like a short circuit)

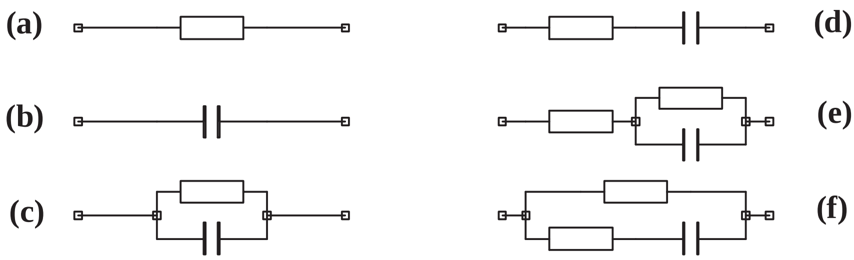


Figure 18

Commonly used networks for Z_1 and Z_2. Networks (b) and (d) are not suitable for Z_2 in single op-amp applications because they do not permit dc feedback.

is $-R_2/R_1$ and these values can be used to check the analytical result of equation **(18)**. The h.f. gain is larger than the l.f. in this case and the response will resemble that of figure 16.

There is a wide range of feedback circuits that one might come across in pole-zero applications. A selection of these is shown in figure 18. In general the feedback element, Z_2, will have to allow dc feedback, the exception being where dc feedback is provided by some other path such as the system shown in figure 5. In all the pole-zero applications, either Z_1 or Z_2 will be a simple resistor - ie, figure 18 (a).

Intrinsic linear frequency response of the op-amp

The op-amp itself is not perfect in terms of frequency response; it has a limited bandwidth. Figure 19 shows the frequency response associated with A_v - the open loop gain - for a typical op-amp. Also shown in figure 19 are two closed loop gain responses for different closed loop gains. Only one of the closed loop phase responses has been shown in order to avoid clutter on the diagram. Key points are

- all the roll-offs follow the open loop curve
- each of the three responses shown exhibit first order behaviour.
- for the open loop response the product of dc gain and -3 dBbandwidth - 300,000 x 10 Hz in this case - is equal to the product of gain and frequency at f_1 - the point where gain is unity (1 x 3 MHz in this case).
- for the closed loop gain cases, the product of dc gain (ie. the gain at a frequency low enough such that gain is independent of frequency) and the - 3dB frequency is a constant equal to the unity gain frequency of the open loop response.

This leads to the important idea

dc gain x -3dB bandwidth = constant = open loop unity gain frequency

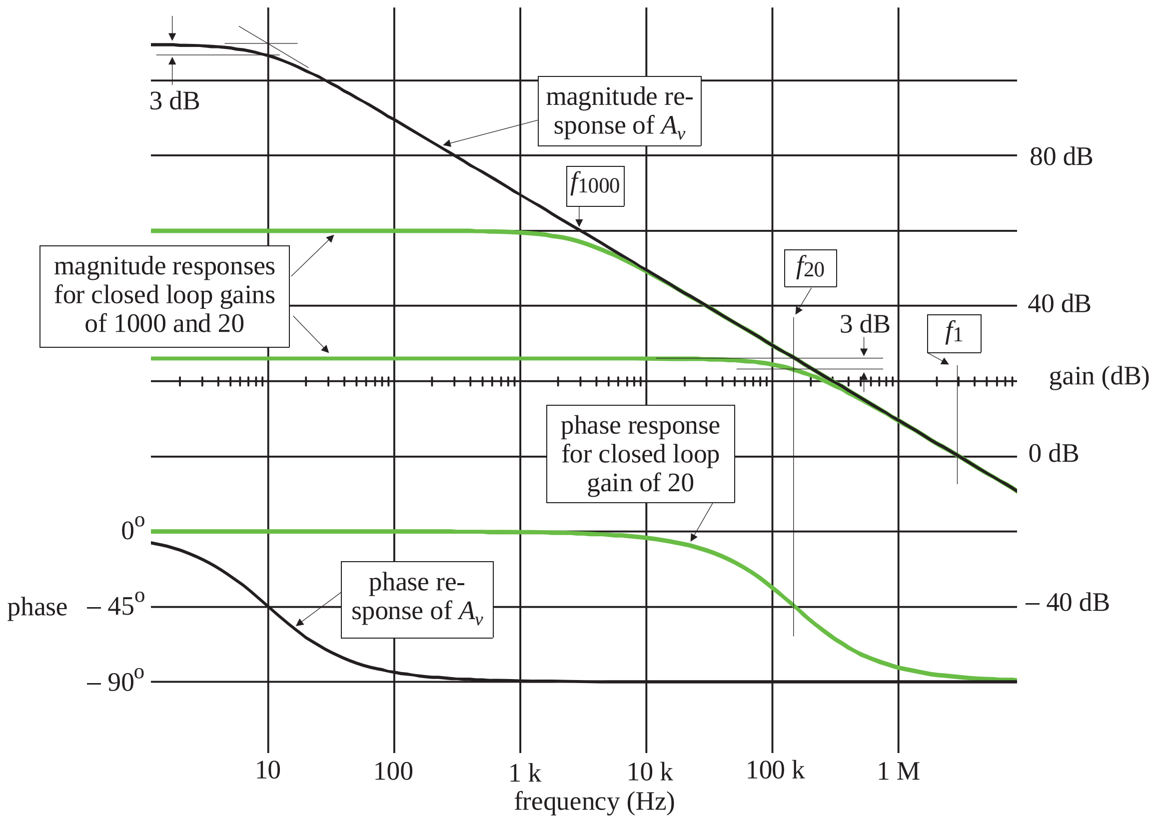


Figure 19

The open loop response of a typical op-amp with two closed loop responses on the same graph. Only one of the closed loop phase responses has been shown.

The ***only*** information provided by manufacturers to describe the linear frequency response behaviour is the "gain - bandwidth" product (*GBP*) or "unity gain frequency". Because the amplifier behaves as a first order system, this parameter and knowledge of circuit gain is all that is required to work out any aspect of linear frequency or time dependent behaviour. For circuits with frequency dependent feedback, the gain-bandwidth product idea can be used when the phase shift due to the feedback circuit is close to zero at the frequency where the circuit high frequency gain meets the op-amp's open loop response. If feedback phase is significant at this point, the circuit must be treated as a second order system (and they will not be dealt with in this module).

Knowing *GBP* (or unity gain frequency) and required circuit gain, the circuit bandwidth can be evaluated. Manufacturers usually quote unity gain frequency in Hz but in circuit design situations it may be necessary to convert this to radians per second. Since the op-amp and op-amp circuits with resistive feedback behave like first order systems, knowledge of frequency domain behaviour also gives knowledge of time domain behaviour.

The key first order relationships of the op-amp itself are between gain and frequency and time and frequency

$$A_v = \frac{A_0}{1 + j\dfrac{\omega}{\omega_0}} \qquad \textbf{(19)}$$

$$\tau = \frac{1}{\omega_0} = \frac{1}{2\pi f_0} \qquad \text{or} \qquad f_0 = \frac{\omega_0}{2\pi} = \frac{1}{2\pi\,\tau} \qquad \textbf{(20)}$$

where A_v is the amplifier gain. A_0 its dc (0 Hz) gain, τ is the system time constant, f_0 is the cyclic open loop corner frequency (in Hz) and ω_0 is the angular open loop corner frequency (in radians per second). For a non-inverting amplifier with feedback that behaves resistively

$$K_v = \frac{K_0}{1 + j\dfrac{\omega}{\omega_K}} \quad \text{where} \quad \omega_K = \frac{GBP}{K_0} \quad \text{and} \quad \tau_K = \frac{1}{\omega_K} \qquad \textbf{(21)}$$

In equation **(21)** *GBP* must be in radians per second. Equation **(21)** could be written in terms of cyclic frequency by changing all ω_K to f_K and using the ω - f relationships of equation **(20)**.

The "gain x bandwidth = constant" rule is true for all non-inverting amplifier circuits built using amplifiers that are described by manufacturers as "unity gain compensated". The intrinsic response of the amplifier is usually not first order - in fact most amplifiers have three first order responses in series, one for each of input stage, voltage gain stage (VAS) and output stage. This is not a problem if over the range of frequency where $|A_v| > 1$, the behaviour is governed by one of the three first order behaviours - usually that associated with the VAS. In a unity gain compensated amplifier, a category that encompasses almost all general purpose amplifiers, the manufacturers deliberately ensure that the VAS response dominates the amplifier response for all $|A_v| > 1$. They do this by deliberately increasing the collector - base capacitance of the VAS transistor to a value of between 10 pF and 30 pF as necessary. The effect of this capacitor is magnified as far as the base circuit of the VAS is concened by an effect known as Miller multiplication.

Miller multiplication is an effect that can be explained by the Miller Transform, a circuit transformation that represents feedback elements connected between input and output of an amplifier as equivalent shunt elements between input and ground and output and ground.

Consider figure 20. In figure 20(a) an amplifier with a parallel *RC* feedback circuit is shown. The amplifier has a terminal voltage gain *A*. The Miller Transformation identifies the effective values of the feedback elements *R* and *C* from the point of view of the source circuit (ie, the circuit that produces v_i) and the load circuit. Effectively this process aims to model the circuit of figure 20(a) with that of figure 20(b) and the Miller transformation finds the C_{iM} and R_{iM} that give the same ratio v_i/i_i for both parts of figure 20.

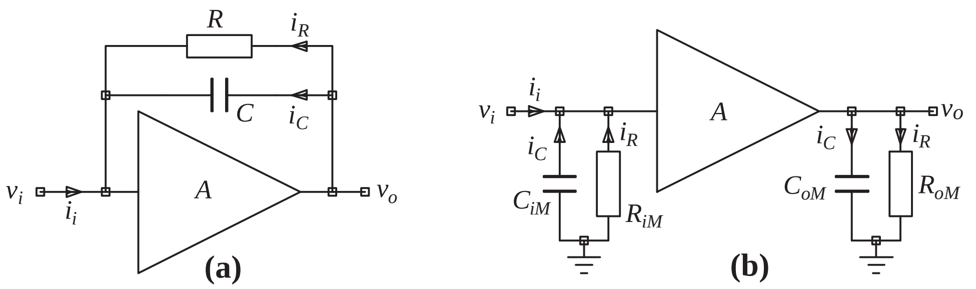


Figure 20

*An amplifier circuit **(a)** with its Miller transformed feedback components **(b)***

In figure 20(a), i_R and i_C are given by

$$i_R = \frac{v_o - v_i}{R} \text{ and } i_C = (v_o - v_i)j\omega C\,.$$

In figure 20(b), i_R and i_C are given by

$$i_R = \frac{0 - v_i}{R_{iM}} = \frac{-v_i}{R_{iM}} \text{ and } i_C = -v_i\, j\omega C\,.$$

For the two circuits to be the same from the source point of view i_R and i_C in figure 20(a) must be the same as the i_R and i_C in figure 20(b). Thus

$$i_R = \frac{v_o - v_i}{R} = -\frac{v_i}{R_{iM}} \qquad \text{or} \qquad \frac{Av_i - v_i}{R} = -\frac{v_i}{R_{iM}}$$

$$\text{so } R_{iM} = \frac{R}{1-A} \tag{22}$$

and

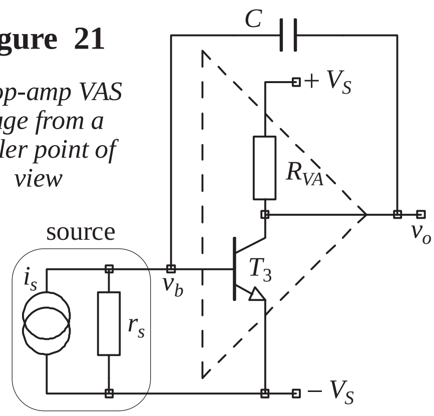
$$i_C = (v_o - v_i)j\omega C = -v_i\, j\omega C_{iM} \qquad \text{or} \qquad (Av_i - v_i)j\omega C = -v_i\, j\omega C_{iM}$$

$$\text{so } C_{iM} = C\,(1-A) \tag{23}$$

This means that as far as the amplifier's signal source is concerned, the feedback impedance is a factor of $(1-A)$ times lower than the component face value. Thus, the effective capacitance seen by the signal source is $(1-A)$ times the actual value of feedback capacitance and this effect is known as Miller multiplication. In the case of the VAS stage, the effective circuit is shown in figure 21. *C* is the total capacitance between collector and base nodes (sometimes called the Miller capacitance). The gain $v_o/v_b \equiv A = -g_{m3}R_{VA}$ which

Figure 21

An op-amp VAS stage from a Miller point of view



is usually a large negative number (typically – 500 to – 50,000). Thus the equivalent circuit seen by the source is as shown in figure 22. If T_3 is a Darlington connected pair, r_{be3} could be several hundred kΩ so the large τ caused by the large effective C gives rise to a low corner frequency.

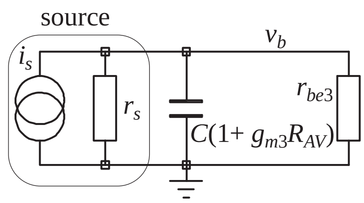


Figure 22

The equivalent circuit seen by the output of the differential input stage (the source) as it looks into the input of the VAS stage

There are also components C_{oM} and R_{oM} between the amplifier output and ground and for completeness these shoul;d be mentioned. These tell us what the feedback elements look like from the amplifier output's point of view. If A is large and negative, these components have effective values that are close to the actual feedback component values. Using an approach similar to that used for the input Miller transformed feedback elements the output elements are

$$R_{oM} = R\frac{A}{A-1} \quad \text{and} \quad C_{oM} = C\frac{A-1}{A}$$

Note that no conditions have been imposed upon A in the foregoing considerations. A could be positive or complex. A moment's thought will reveal that this opens up some interesting possibilities. We assume here that A is real, negative and large. Note also that R_{oM} and C_{oM} represent the impedance of the feedback components as seen by the op-amp output; they do not represent the output impedance of the whole circuit.

[Where a feedback resistance exists, its effective value from a source point of view is its actual value divided by the factor $(1-A)$. One example of this effect is a simple op-amp inverting amplifier circuit. In such a circuit there is a feedback resistor, R_F, but usually no capacitor. The feedback resistor is connected between the output and the inverting input. From the point of view of the signal source the input impedance of the circuit is usually taken as the resistor, R_1, between the source and the inverting input - but this assumes infinite op-amp gain. As far as the input circuit is concerned, R_F appears as a resistor between inverting input and ground with a value of $R_F/(1-A)$. For typical values of A and R_F - say 3×10^5 and 100 kΩ - $R_F/(1-A) = 0.3$ Ω, a very low value. This is another way of looking at the notion of a virtual earth.]

Non-linear effects - slew rate limiting

A second frequency dependent effect that is of some importance in applications where a big output voltage swing at high frequencies is required is known as "slew rate limiting". Slew rate limiting is a process that limits the maximum rate of change of voltage that the op-amp can support at its output and is usually specified by manufacturers in terms of V μs^{-1}. Slew rate limiting is a non-linear process - its effect depends on the magnitude of the signal as well as on the frequency of the signal. Slew rate limiting is non linear because when it occurs, the ratio $\Delta v_o/\Delta v_i$ is a function of v_i - ie, gain depends upon signal level - a classic sign of a non-linear process.

Slew rate limiting is caused by the interaction between the collector - base capacitance, C_{cb}, of the voltage gain (VAS) transistor and the current sources that drive the collector and base nodes of that transistor. The fact that manufacturers artificially increase the value of C_{cb} of the VAS transistor in compensated amplifiers makes slew rate limiting more of a problem for them. Generally speaking, the less compensation an amplifier has, the better will be its slew rate capabilities . . . . but it might have stability problems in low gain applications. These two competing effects provide a good example of an engineering design tradeoff; one desirable system property wants a parameter to be large, another wants it to be small.

From an application circuit design point of view one would usually specify a minimum slew rate requirement for any amplifiers in the design based on knowledge of the worst case signal the amplifier must handle. From a slew rate point of view the worst case is the largest rate of change of voltage that will ever occur in the signal at the op-amp output.

Rectangular waveshapes

Rectangular waveshapes with ideal (instantaneous) rising and falling edges applied to the op-amp input will produce exponential responses at the output because of the op-amp's first order frequency response behaviour. If the rising and falling edges of the input waveform are not ideal the output response will have a lower maximum rate of change than it would for an ideal input so the ideal input provides a good working worst case estimate. A positive going step input to an amplifier circuit described by equation **(21)** would yield an output response of the form

$$v_o(t) = v_{span}\left(1 - e^{-\frac{t}{\tau_K}}\right)$$

where v_{span} is the difference between starting and aiming levels of the rising exponential. The maximum rate of change of this function is v_{span}/τ_K and occurs at the start of the exponential shape. This maximum rate of change of v_o is a function of τ_K and hence of K_0.

Sinusoidal waveshapes

A sinusoid at the output of an amplifier circuit is

$$v_o(t) = v_P \sin \omega t$$

where v_P is the amplitude of the sinusoid. The largest rate of change of $v_o(t)$ is $v_P\omega$.

Solving problems

The design problems associated with slew rate limiting fall usually into two types - both different forms of the same problem.

One is to identify a suitable op-amp for a particular application based on knowledge of the application requirements and published op-amp specifications. Here the objective is to identify the biggest rate of change of voltage in the required output signal and then search for an amplifier that can meet that requirement.

The other is to identify what limitations in terms of frequency and/or amplitude of output signal will be imposed on a design by the use of a particular op-amp. These problems can be expressed in a number of ways but they all involve equating a maximum rate of change of signal voltage at the amplifier output to the slew rate.