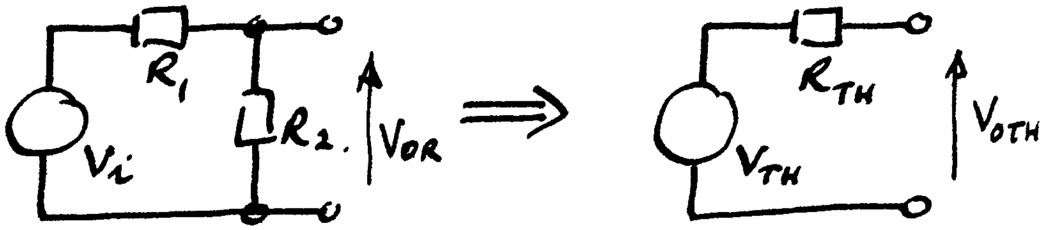
EEE103 / EEE121 / EEE141 Problem Sheet Solutions

Background Knowledge

Q1



real circuit ⟹ Thevenin equivalent.

Need to find the R_{TH} + V_{TH} that will make the Thevenin equivalent indistinguishable from the real cct .... ie

output voltage of real = output voltage thevenin

short circuit output current of real = short circuit output current of thevenin

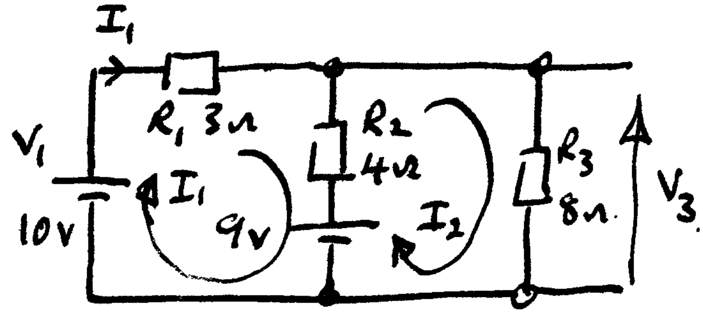
$$V_{OR} = V_i \frac{R_2}{R_1+R_2} = V_{OTH} \text{ for equivalence } = V_{TH}$$

$$I_{SCR} = V_i/R_1 = V_{TH}/R_{TH} \text{ for equivalence}$$

$$\text{so } R_{TH} = \frac{V_{TH}R_1}{V_i} = \frac{R_1}{V_i} . V_i \frac{R_2}{R_1+R_2} = R_1 \| R_2 .$$

$$\text{ie } V_{TH} = V_i \frac{R_2}{R_1+R_2} \text{ and } R_{TH} = \frac{R_1R_2}{R_1+R_2} .$$

Q2. Firstly using loops:



$$10 = I_1R_1 + (I_1 - I_2)R_2 + 9 \qquad (1)$$

$$9 = R_2(I_2 - I_1) + R_3 I_2 \quad \text{—— (2)}$$

$$V_3 = R_3 I_2 \quad \text{—— (3)}$$

expanding (1)

$$10 = 3I_1 + 4I_1 - 4I_2 + 9$$

or $1 = 7I_1 - 4I_2$ —— (4)

expanding (2)

$$9 = 4I_2 - 4I_1 + 8I_2$$

or $9 = 12I_2 - 4I_1$ —— (5)

eliminating I_2 from (4) + (5) gives.

$$9 = 12\left[\frac{7I_1 - 1}{4}\right] - 4I_1 = 21I_1 - 3 - 4I_1$$

or $I_1 = 12/17 = \underline{\underline{0 \cdot 706\,A}}$.

using (4), $1 = 7 \times \frac{12}{17} - 4I_2$ or $I_2 = 0.985A$.

and using (3), $V_3 = 8I_2 = \underline{\underline{7 \cdot 88\,V}}$.

Using superposition to find V_3 .....

$$V_{3(10v)} = V_1 \cdot \frac{R_2 \| R_3}{R_1 + R_2 \| R_3} = 10 \cdot \frac{32/12}{3 + 32/12} = \frac{10 \cdot 8/3}{17/3}$$

$$= 80/17\ V.$$

$$V_{3(9v)} = V_2 \frac{R_1 \| R_3}{R_2 + R_1 \| R_3} = 9 \cdot \frac{24/11}{4 + 24/11} = 9 \cdot \frac{24/11}{68/11}$$

$$= 54/17\ V$$

$$V_{3\,TOT} = V_{3(10)} + V_{3(9)} = \frac{134}{17} V = \underline{\underline{7 \cdot 88\ V}}.$$

using superposition to find I_1 ....

$$I_{1(10)} = V_1/(R_1 + R_2 \| R_3) = \frac{10}{3 + 8/3} = \frac{30}{17}\ A.$$

$$I_{1(9)} = -V_3/R_1 = -\frac{1}{R_1} \cdot \frac{9 \cdot 24/11}{4 + 24/11} = -\frac{1}{3} \cdot 9 \cdot \frac{6}{17}$$

$$= -18/17.$$

$$\therefore I_{1\,TOT} = I_{1(10)} + I_{1(9)} = \frac{30}{17} - \frac{18}{17} = \underline{\underline{0.706\ A}}.$$

To find Norton equivalent ....

(i) put short cct across output terminals and calculate current through it .....

$$I_{sc(10V)} = \frac{10}{3} = 3.33\ A.$$

$$I_{sc(9V)} = \frac{9}{4} = 2.25\ A.$$

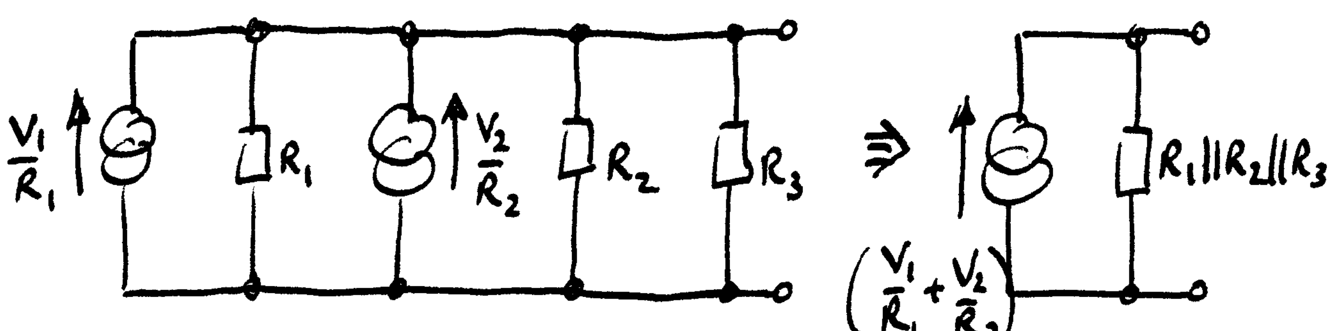
$$\therefore I_{sc\,TOT} = \frac{10}{3} + \frac{9}{4} = \frac{67}{12} = \underline{5.58\ A}.$$

$\therefore$ The Norton current source is 5.58 A.

(ii) The Norton parallel resistance is V_3/I_N.

$$= \frac{134/17}{67/12} = \frac{12 \times 134}{67 \times 17} = \frac{24}{17} = \underline{\underline{1.41\ \Omega}}.$$

One could also have transformed the limbs of the original circuit... and then summed ....



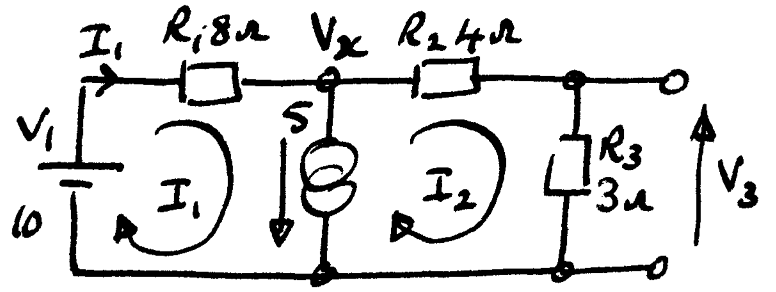
to find the value of V_2 that would make $I_1 = 0$, one can make use of the superposition process at the top of page 3 with 9V replaced by V_2 ....

$$I_{1TOT} = I_{1(10)} + I_{1(V_2)} = \frac{30}{17} - V_2 \cdot \frac{2}{17}$$

and $I_{1(TOT)} = 0$ is required .... so

$$\frac{30}{17} - \frac{V_2 \cdot 2}{17} = 0 \quad \text{or} \quad V_2 = \frac{30}{2} = \underline{\underline{15V}}.$$

Q3 Using loops... it is necessary to define a variable V_x for the unknown node voltage ...



$$10 = I_1R_1 + V_x \quad \text{——— (1)}$$
$$V_x = I_2R_2 + I_2R_3 \quad \text{——— (2)}$$
$$I_1 - I_2 = 5 \quad \text{——— (3)}$$

eliminating V_x from (1) and (2) ...

$$10 = I_1R_1 + I_2R_2 + I_2R_3$$
$$= 8I_1 + 7I_2$$

and using (3) to eliminate I_2 ...

$$= 8I_1 + 7(I_1 - 5) = 15I_1 - 35.$$

$$\text{or } I_1 = \frac{35+10}{15} = \frac{45}{15} = \underline{\underline{3A}}.$$

using (3), $\quad I_2 = -5 + I_1 = \quad -2A$

$$\therefore V_3 = I_2R_3 = \underline{\underline{-6V}}.$$

Using superposition to find I_1 ...

$$I_{1(10)} = 10/(8+4+3) = 2/3\,A.$$

$$I_{1(5A)} = -\frac{V_x}{R_1} = -\frac{(-5(R_2+R_3)\|R_1)}{R_1}$$

$$= \frac{5\,(R_2+R_3)}{R_1+R_2+R_3} = \frac{5\times 7}{15} = \frac{7}{3}.$$

$$\therefore I_{1TOT} = I_{1(10)} + I_{1(5A)} = \frac{2}{3}+\frac{7}{3} = \underline{\underline{3A}}.$$

to find V_3 ...

$$V_{3(10)} = 10.\frac{R_3}{R_1+R_2+R_3} = 10.\frac{3}{15} = 2V.$$

$$V_{3(5)} = V_x.\frac{R_3}{R_2+R_3} = -5(R_2+R_3)\|R_1.\frac{R_3}{R_2+R_3}$$

$$= -5.\frac{56}{15}.\frac{3}{7} = -8V.$$

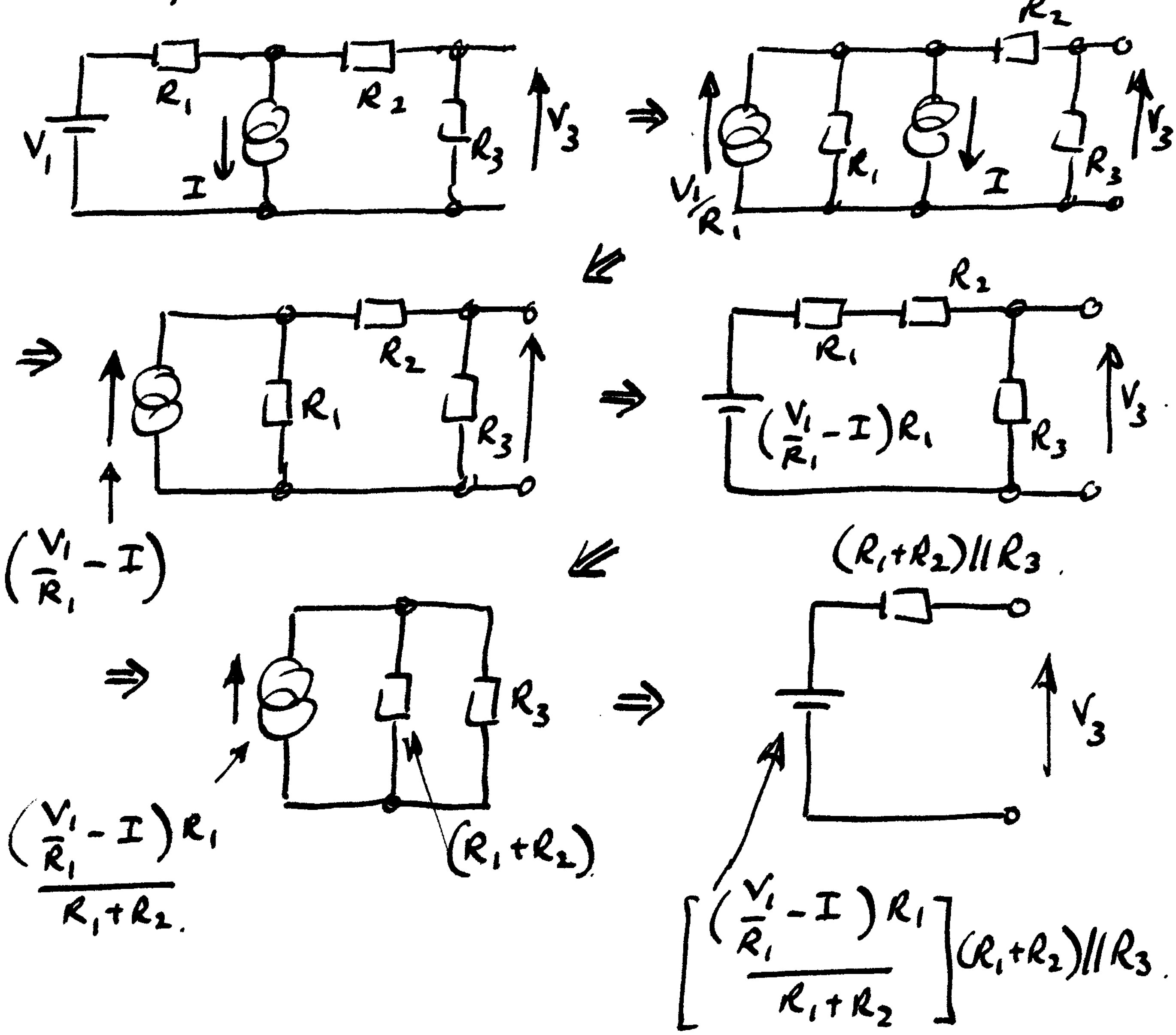
$$V_{3TOT} = V_{3(10)} + V_{3(5A)} = 2-8 = \underline{\underline{-6V}}.$$

For the Thevenin equivalent circuit, V_{TH} is V_3 (by definition) and to find R_{TH}, either look into V_3 terminals with V_1 replaced by 0Ω and I by $\infty\ \Omega$ and work out resistance or work out the current that would flow through a short circuit placed across the V_3 terminals and use $R_{TH} = V_3/I_{sc}$.

$$V_{TH} = \underline{\underline{-6V}}.$$

$$R_{TH} = R_3 \| (R_1+R_2) = \frac{36}{15} = \underline{\underline{2.4\Omega}}.$$

or yet another possibility is to do successive transformations ....



This method is a bit laborious but gives excellent transformation practice.

To find I that will make $V_3 = 0$, use superposition V_3 approach on page 5 and replace 5 by I ...

$$V_{3(10)} + V_{3(I)} = 0 = 2V + \frac{\left(-I(R_2+R_3)\|R_1\right)R_3}{R_2+R_3}$$

$$\text{or } 2 = I\,\frac{R_1R_3}{R_1+R_2+R_3} = I \cdot \frac{24}{15} \qquad \therefore \underline{\underline{I = 1.25A}}$$

Q4 (i) everything has units of current except for the I_5/R_6 term

(ii) the common unit is volts. The $I_4(R_3+1)$ and the R_3 terms are wrong.

(iii) is correct; both sides have units of R.

(iv) the unit on both sides is V. All the $j\omega$ terms are dimensionless (and hence correct) except for the last one, $j\omega C_2 R_1 R_2$, that has units of R and is incorrect. [remember ω has units of 1/time, CR has units of time, j is dimensionless.]

(v) the $j\omega(C_1+C_2)R^2$ term has units of R and should be dimensionless.

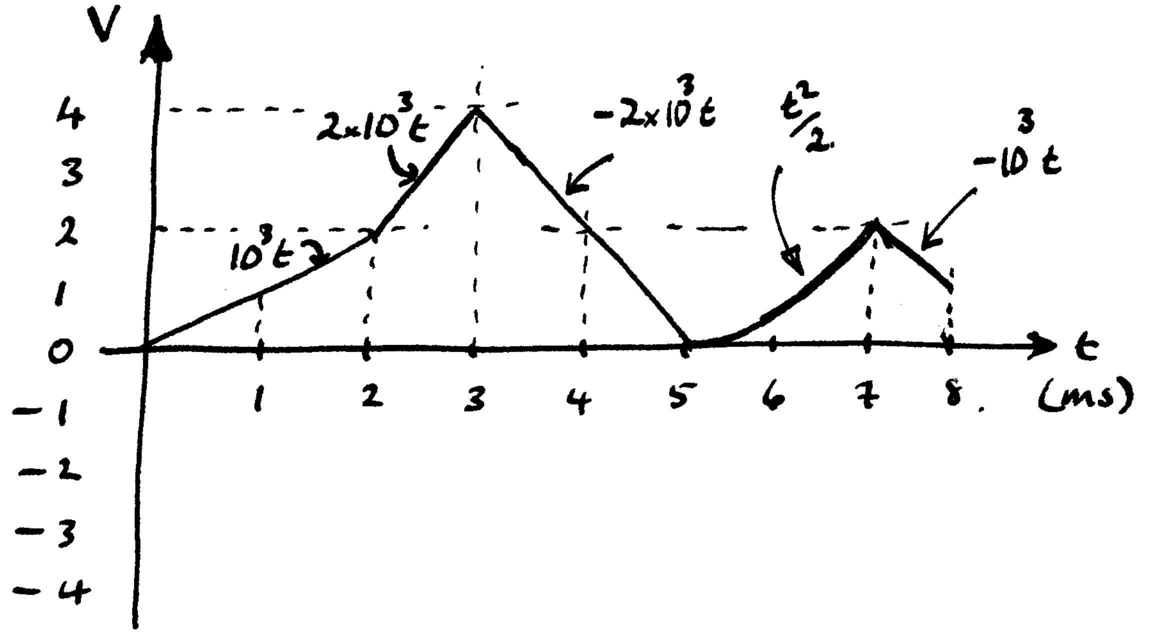
(vi) the $j\omega L$ term has units of R and should be dimensionless.

(vii) is correct; Z is impedance with units of R, each term in numerator of right hand side has units of R, each term in r.h.s. denominator is dimensionless.

Q5 The key relationship here is

$$V_c = \frac{1}{C}\int I\,dt.$$

and it is quite helpful to remember that $\int I\,dt$ = charge. In the absence of impulsive currents, there are no instantaneous changes in charge and hence no sudden jumps in voltage it is often easier in this type of problem to move the time origin to a convenient location for each piecewise linear section.



between 1 + 2ms, $V = \frac{1}{C}\int 1\text{mA}\,dt = 10^3 t$

" 2 + 3ms, $V = \frac{1}{C}\int 2\text{mA}\,dt = 2\times10^3 t$

" 3 + 5ms, $V = \frac{1}{C}\int -2\text{mA}\,dt = -2\times10^3 t$

" 5 + 7ms, $V = \frac{1}{C}\int t\,dt = \frac{t^2}{2}$

" 7 + 8ms, $V = \frac{1}{C}\int -1\text{mA}\,dt = -10^3 t$

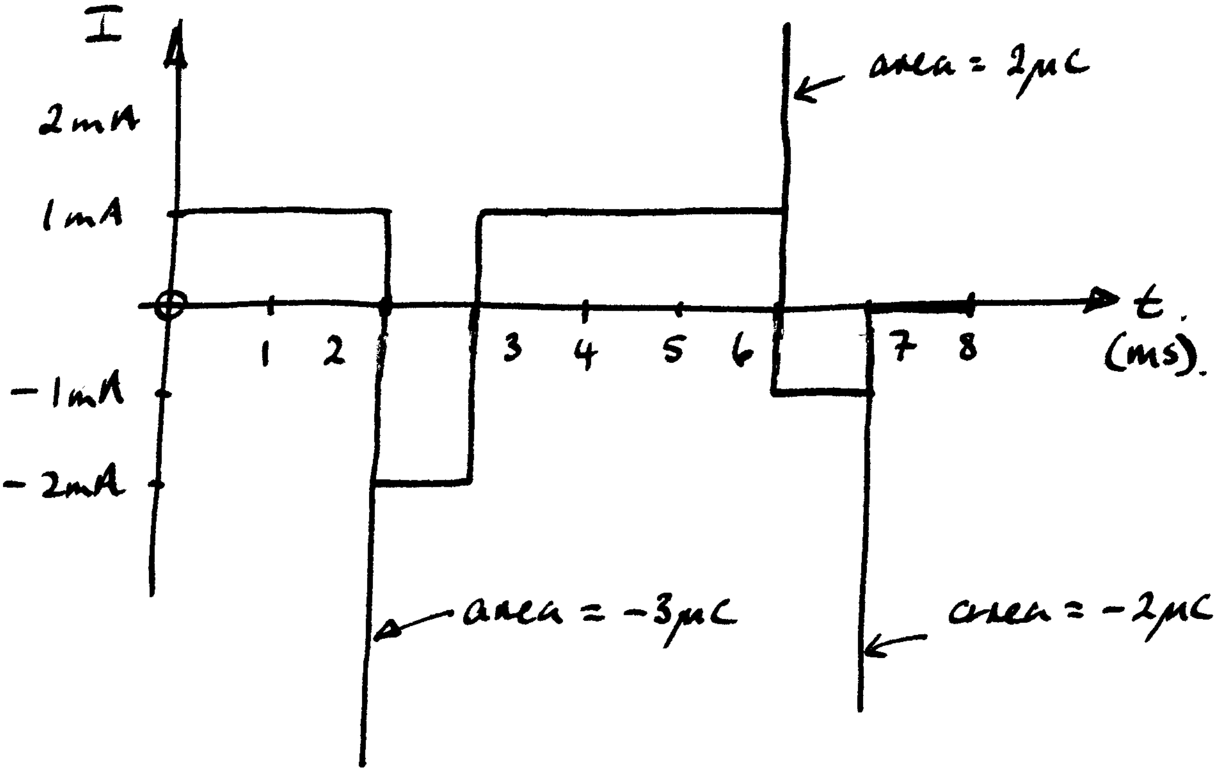
charge at end can be worked out either by adding up the total area under the I-t graph given (which is essentially what the integrating process does) or by working out the charge necessary to support the 1V final voltage

ans = $\underline{1\mu C}$.

Q6. The key relationship here is

$$I = C\frac{dV}{dt}.$$

and the only difficulties lie in the places where $\left|\frac{dV}{dt}\right| = \infty$. When this happens, a charge appropriate for the ΔV must enter C in zero time leading to a current pulse that is infinitely high and infinitely thin. The only thing that is defined about the pulse is its area ... which, of course, is equal to the charge change caused by the voltage change.



0ms → 2ms, $\frac{dV}{dt} = 10^3$, $I = 1mA$.
2ms → 3ms, " $= -2\times10^3$, $I = -2mA$
3ms → 6ms, " $= 10^3$, $I = 1mA$
6ms → 7ms, " $= -10^{-3}$, $I = -1mA$.
7ms → " $= 0$ $I = 0$.