EEE118: Electronic Devices and Circuits

Lecture II

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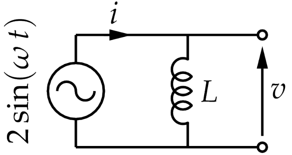
Last Lecture: Review

- Stated the Aims and Objectives of the course
  How electronic devices (diodes, transistors et al. work in circuits
- Introduced some Circuit Terminology (Voltage, Current, Node, Branch)
- Introduced Engineering Units
  units use powers of three. 100 nA, 1 uA, 10 uA, 100 uA, 1 mA, 10 mA etc.
- Discussed two Passive Components, their physical construction (Resistors and Capacitors), relative price and performance.
- Considered the relationship between current and voltage in R & C in the time domain.

Outline

Inductor Construction and Technology

- Inductors are two terminal electrical components which store energy in a magnetic field.
- Composed of one or more electrical conductors wound onto a ring of magnetic material.
- Or one or more insulated electrical conductors wound onto plastic/cardboard former and possibly slid onto an iron or ferrite core to form a magnetic circuit.
- Several inductors may be wound so the magnetic flux is coupled between them to form a transformer.



$$i = \frac{1}{L}\int v\,\mathrm{d}t.$$

$$i = \frac{1}{L}\left[\frac{-\cos(\omega\, t)}{\omega}\right]$$

$$i = \frac{\sin(\omega\, t - \pi/2)}{\omega\, L}$$

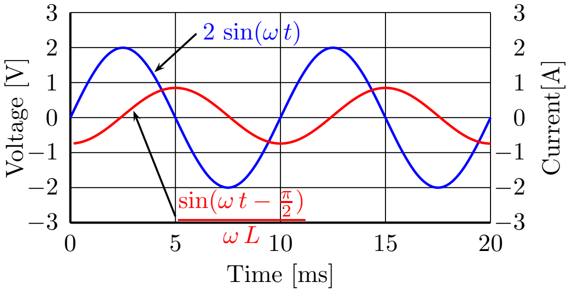
$$x_L = j\,\omega\, L$$

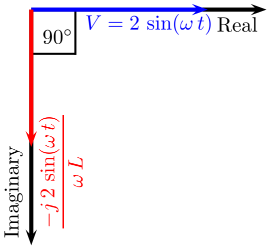
$$i = v/x_L$$

$$i = \frac{2\,\sin(\omega\, t)}{j\,\omega\, L}$$

$$1/j = -j$$

$$i = \frac{-j\,2\,\sin(\omega\, t)}{\omega\, L}$$

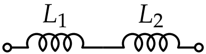
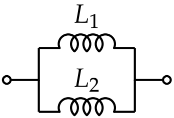




Red: Inductor Current, Blue: Inductor Voltage. i lags v by 90°.

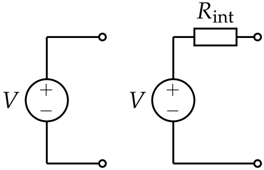
Simple Inductor Circuits

Voltage can be applied and current can flow, like resistors, but ideal inductors do not dissipate power because the phase of the current lags the voltage by 90°. $P = I\,V\cos(\phi)$ Where ϕ is the phase angle between voltage and current. $cos(\phi)$ is the *power factor*.

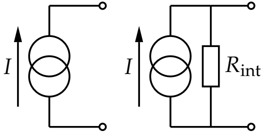
Series		$L = L_1 + L_2$
Parallel		$L = \frac{L_1 \cdot L_2}{L_1 + L_2}$

Voltage and Current Sources

An ideal voltage source is a two terminal circuit element supplying a fixed voltage and having zero internal resistance. A real voltage source can only supply a finite current and behaves as an ideal source with a resistance in series. It has non-zero internal resistance *in series* with the ideal voltage source.

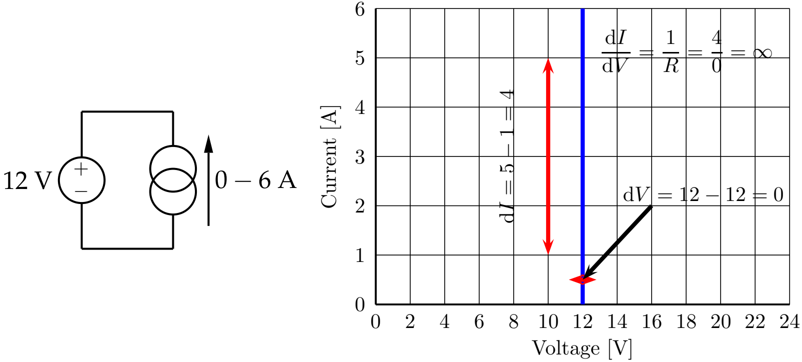


An ideal current source is a two terminal circuit element supplying a fixed current and having infinite internal resistance. A real current source can only supply the specified current over a range of terminal voltages. It has a finite internal resistance *in parallel* with the ideal current source.



Voltage Source Internal Resistance

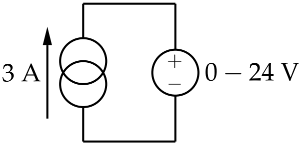
What is the internal resistance of a perfect voltage source? Force a known current into a perfect voltage source and observe the change in voltage, then use Ohm's law to find the internal resistance.

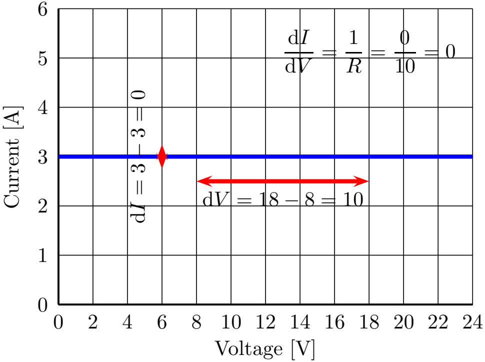


$\frac{1}{\infty} = 0\ \Omega$, so the internal resistance of a perfect voltage source is zero.

Current Source Internal Resistance

What is the internal resistance of a perfect current source? Force a known voltage across a perfect current source and observe the change in current, then use Ohm's law to find the internal resistance.



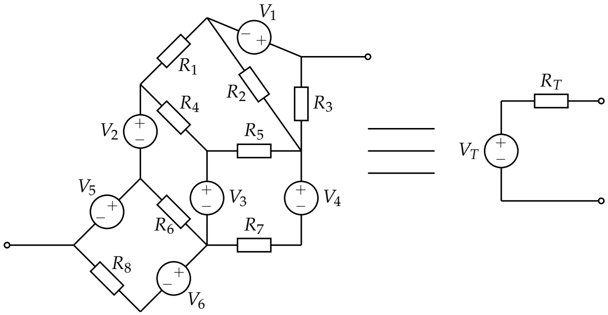


$\frac{?}{0} = \infty\ \Omega$, so the internal resistance of a perfect current source is infinite.

Thévenin

Theorem

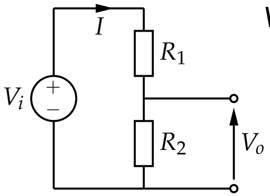
Any network of resistance elements and energy sources can be replace by a series combination of an ideal voltage source V_T and a resistance R_T where V_T is the open-circuit voltage of the circuit and R_T is the ratio of the open circuit voltage to the short circuit current.



Thévenin Method

- Find V_T by measurement or calculation of the voltage across the nodes of interest without anything connected (open-circuit)
- Find by measurement or calculation the current (I_{sc}) that flows when the nodes of interest are connected together (short-circuit).
- Divide V_T by I_{sc} to yield R_T.

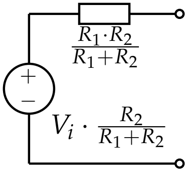
For example,



$$V_T = V_o = V_i \cdot \frac{R_2}{R_1 + R_2}$$

$$I_{sc} = \frac{V_i}{R_1}$$

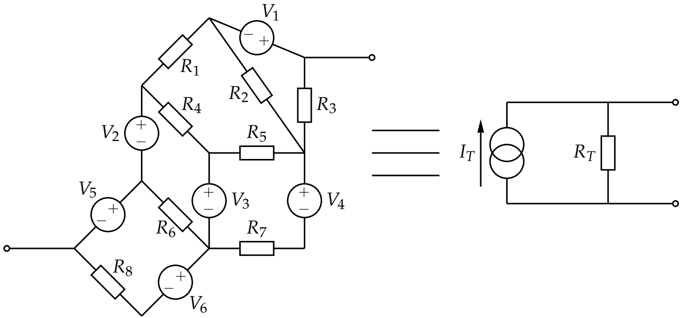
$$R_T = \frac{R_1\,R_2}{R_1 + R_2}$$



Norton

Theorem

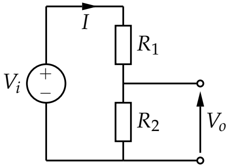
Any network of resistance elements and energy sources can be replace by a parallel combination of an ideal current source I_T and a resistance R_T where I_T is the shot-circuit current of the circuit and R_T is the ratio of the open circuit voltage to the short circuit current.



Norton Method

- Find I_N by measurement or calculation the current that flows from one node to the other when they are short-circuit (connected together)
- Find by measurement or calculation the voltage (V_{oc}) that appears across the nodes of interest when nothing is connected between them (open-circuit)
- Divide V_{oc} by I_N to yield R_N.

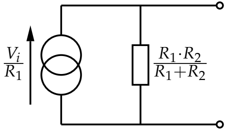
For example,



$$I_N = \frac{V_i}{R_1}$$

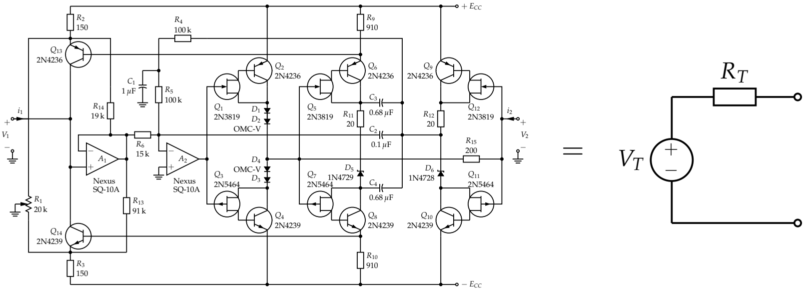
$$V_{oc} = V_i \cdot \frac{R_2}{R_1 + R_2}$$

$$R_N = \frac{R_1\,R_2}{R_1 + R_2}$$



Source Transformations Summary

Active and passive circuits can be treated as a “black box” and thought of in terms of their Thévenin equivalent voltage and series resistance or Norton equivalent current and parallel resistance.



Superposition

Theorem

If a circuit consists of linear components (or components that can be considered linear over a small range of voltage and current), the combined effect of several energy sources on the circuit is equal to the sum of the effects of each source acting alone.

The theorem implies that the sources should be considered independently, but does not say what to do with the ones we are not considering!

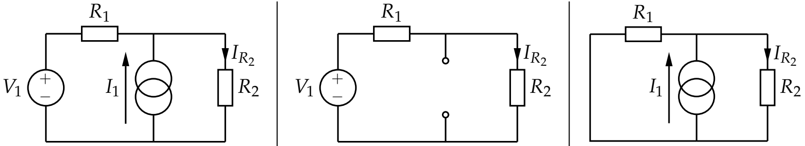
Consider the internal resistance of perfect voltage and current sources (look back at the earlier slides).

Current sources are replaced by an infinite resistance open circuit.

Voltage sources are replaced by zero resistance short circuit.

Superposition Example

Find the contribution of each source to the current flowing in R_2.



Both Sources	Voltage Source	Current Source
??	$I_{R_2} = V_1 \cdot \frac{1}{R_1+R_2}$	$I_{R_2} = I_1 \cdot \frac{R_1}{R_1+R_2}$

Also by inspection the two expressions for I_2 have current flowing in the same direction so they are summed to yield,

$$I_{R_2} = V_1 \cdot \frac{1}{R_1 + R_2} + I_1 \cdot \frac{R_1}{R_1 + R_2}$$

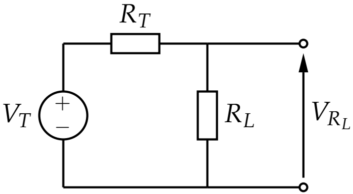
See Smith, R. J., and Dorf, R. C., *Circuits Devices and Systems* 5th ed., Wiley, 1992, pp. 56, dd. 621.3

Power Transfer

Consider an imperfect voltage source, where the internal resistance is not zero. Is there an optimum resistance to transfer the maximum power from the source into the circuit?

Two methods,

1. Trial and error with example numbers
2. Mathematical derivation

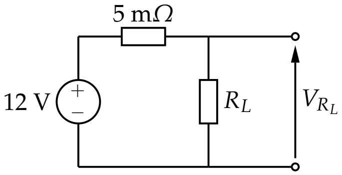


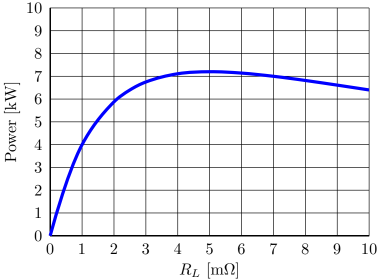
$$P = I\,V \text{ and } P = \frac{V^2}{R} \text{ and } P = I^2\,R$$

Trial and Error

Let R_L be,

1. 2.5 mΩ
   $V_{R_L} = 12 \cdot \frac{2.5}{2.5+5}\ \mathrm{m\Omega} = 4\ \mathrm{V}.$
   $P = \frac{V^2}{R} = \frac{4^2}{2.5\times10^{-3}} = 6.4\ \mathrm{kW}$
2. 5 mΩ
   $V_{R_L} = 12 \cdot \frac{5}{5+5}\ \mathrm{m\Omega} = 6\ \mathrm{V}.$
   $P = \frac{V^2}{R} = \frac{6^2}{5\times10^{-3}} = 7.2\ \mathrm{kW}$
3. 7.5 mΩ
   $V_{R_L} = 12 \cdot \frac{7.5}{7.5+5}\ \mathrm{m\Omega} = 7.2\ \mathrm{V}.$
   $P = \frac{V^2}{R} = \frac{7.2^2}{7.5\times10^{-3}} = 6.9\ \mathrm{kW}$





The maximum power transfer seems to occur when $R_L = R_T$.
A more rigorous approach is desirable however.

Derivation of Maximum Power Transfer Condition

$$P_{R_L} = \frac{V_{R_L}^2}{R_L} \quad V_{R_L} = \frac{V_T\, R_L}{R_L + R_T}$$

Substituting,

$$P_{R_L} = \frac{V_T^2\, R_L}{(R_L + R_T)^2}$$

Differentiating with respect to R_L,

$$\frac{\mathrm{d}P_{R_L}}{\mathrm{d}R_L} = \frac{V_T^2}{(R_L + R_T)^2} - \frac{2\, V_T^2\, R_L}{(R_L + R_T)^3}$$

Set equal to zero (to find the turning point) and solve for R_L,

$$R_L = R_T$$

Review

- Finished discussed of Passive Components with inductors their physical construction, relative price and performance.
- Considered perfect and imperfect voltage and current sources
- **Perfect current sources have infinite parallel resistance**
- **Perfect voltage sources have zero series resistance**.
- Introduced the Thévanin and Norton theorems of source transformation. And gave a simple example of each.
- Introduced the Superposition theorem and gave a simple example.
- Considered the conditions required for maximum power transfer from a Thévanin source ($\mathrm{R}_L = R_T$). This result will be used again in EEE225 when studding electronic noise. Could you derive for Norton on your own?