Operational Amplifiers

These notes introduce operational amplifiers. Classical feedback systems are considered briefly and comparisons are drawn with operational amplifiers. The necessity to supply d.c. power to the opamp is discussed before analysis of several common operational amplifier circuits is presented both using an infinite open loop gain approach and considering a finite gain defect. This discussion leads directly into the discussion of transistor and operational amplifier circuits in EEE225.

Operational amplifiers are the most commonly used analogue "building block". They have been around since the 1930s, the first were constructed using thermionic valves, they were big, heavy and power hungry - about the size of a briefcase in fact. For more details on the background of opamps you'll need to do some digging but a profitable place to start is Jung, W., "The Opamp Applications Handbook", Newnes, 2004. The first opamps formed the basis of analogue computers. Early and modern operational amplifiers differ somewhat in various aspects so here we limit ourselves to modern opamps (i.e. 1968 onwards after the design of the μA741). Modern opamps are designed to have,

- A differential input.
- very high input resistance ($> 10^9\ \Omega$).
- very low output resistance ($< 50\ \Omega$).
- very high gain ($> 10^5$)

The opamp relies on feedback to operate effectively as an amplifier and it can be considered in terms of a classical feedback system.

Classical Feedback Systems

The classical feedback system is shown in Fig. 1. It is possible to use the classical feedback system to see why the properties of the opamp listed above are desirable.

If the output voltage is v_o, a fraction, $H\,v_o$ is fed back to the input stage where it is subtracted from v_i. This leaves $(v_i - H\,v_o)$ at the input of the gain stage G, so

$$v_o = G\,(v_i - H\,v_o) \tag{1}$$

$$\frac{v_o}{v_i} = \text{ system gain } = \frac{G}{1 + G\,H} \tag{2}$$

If G is very large, then $G\,H >> 1$ and

$$\frac{v_o}{v_i} \approx \frac{G}{G\,H} = \frac{1}{H} \tag{3}$$

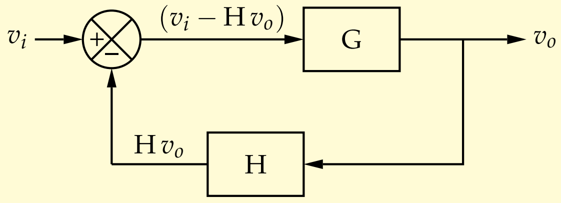
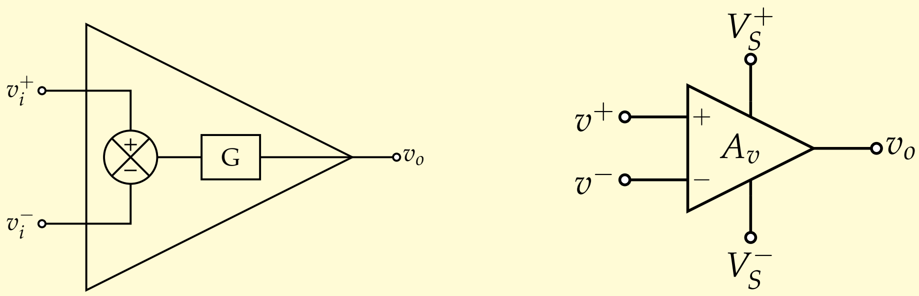


Figure 1: A classical feedback system.

This is a very useful result because it tells the designer that if G is made large enough, system gain is dependent only on the feedback fraction H. H is usually defined by well behaved components – resistors and capacitors – although in this module only resistors will be used.

The Opamp

The opamp integrates two parts of this classical feedback system shown in Fig. 2a.



(a) Opamp parts of a feedback system. (b) Opamp drawn *with* power pins.

Figure 2: Representations of the opamp.

- The input resistances must be high so that the v^- input does not affect the network that defines H and so that the v^+ input does not affect the signal source.
- The output resistance must be low so that the system can drive a load without v_o being affected and so that the system can drive the network defining H without being affected.
- The reason for the differential input and the high gain are evident from the (3). We want H, which is defined by resistors under our control, to set the gain of the amplifier.

- The opamp is usually drawn as shown in Fig. 2b.
- V_s^+ and V_s^- are the power supplies. They are often not included on circuti diagrams but must be connected in the real circuti. v_o cannot move outside the range $V_s^+ > v_o > V_s^-$.
- v^+ is called the "non-inverting" input of the opamp. It is identified by a "+" next to the input line, inside the opamp triange.
- v^- is called the "inverting" input of the opamp. It is identified by a "-" next to the input line, inside the opamp triange.
- the output, v_o, comes from the point of the amplifier symbol.
- A_v is the voltage gain (equivalent of G) which relates the output and intpu by the opamp equation. A_v operates on the *difference* between v^+ and v^- to produce v_o.

$$v_o = A_v \left(v^+ - v^-\right) \tag{4}$$

Opamp Circuits

There are many different circuits that are used with opamps but these are two that are far more common than any others "non-invertin amplifier" and "inverting amplifier".

Non-Inverting Amplifier

When designing an opamp circuit it is usual to initially assume that $A_v \to \infty$. This means that the circuit behaviou is completely controlled by the feedback. If $A_v \to \infty$, for finite v_o, $v^+ \approx v^-$ and this makes working out the circuit behaviour quite straightforward.

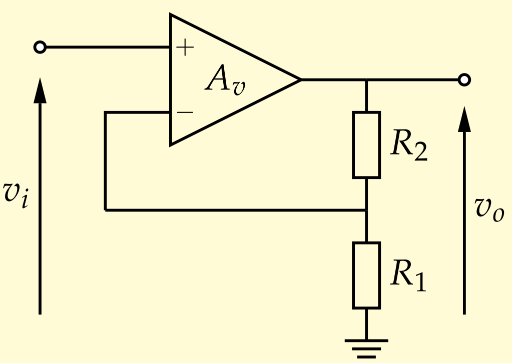


Figure 3: Non-Inverting Circuit.

$$v^- = v_o \frac{R_1}{R_1 + R_2} \text{ by potential division} \tag{5}$$

$$v^+ = v_i \text{ connected by a wire} \tag{6}$$

$$\therefore \text{ if } A_v \to \infty,\ v^+ \approx v^- \tag{7}$$

$$\text{substituting yields } v_i = v_o \frac{R_1}{R_1 + R_2} \tag{8}$$

$$\text{or } \frac{v_o}{v_i} = \frac{R_1 + R_2}{R_1} \tag{9}$$

Notice that the feedback is returned to the input at the "inverting" (v^-) input.

Inverting Amplifier

In the inverting amplifier connection v^+ is grounded and v_i is applied to R_1. Again if $A_v \to \infty$, $v^+ \approx v^-$ and since v^+ is connected to zero v^- must also be very close to zero. The v^- node in this case is called a "virtual earth" because the potential is always close to zero bat the node is not actually connected to zero. The virtual earth exists because of the negative feedback arrangement in this circuit, because A_v is very large and because the opamp subtracts its inputs i.e. obeys the opamp equation. All of these are required for the virtual earth to exist. To work out the gain, start by summing currents at the v^- node...

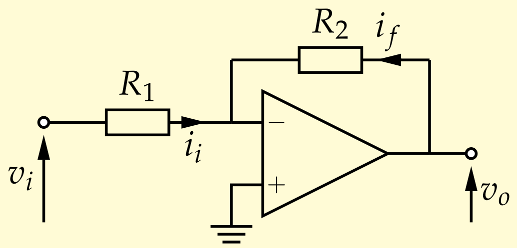


Figure 4: Inverting Circuit.

$$i_i + i_f = 0 \tag{10}$$

substituting via Ohm's law

$$\frac{v_i - v^-}{R_1} + \frac{v_o - v^-}{R_2} = 0 \tag{11}$$

and since $v^- = 0$

$$\frac{v_i}{R_1} + \frac{v_o}{R_2} = 0 \tag{12}$$

transposing for voltage gain

$$\frac{v_o}{v_i} = -\frac{R_2}{R_1} \tag{13}$$

Notice the "-" sign. This means that the signal is inverted (i.e. the phase is shifted by 180°) as well as being amplified. Two inverting amplifiers in series would give rise to an overall non-inverting amplifier – the first stage would invert the signal and the second would invert it back to its original phase.

The Effects of Finite Open Loop Gain

Very occasionally it may be necessary for a designer to estimate the effect of finite opamp open loop gain on the overall circuit gain.

Non-Inverting Amplifier

When considering the effects of finite gain the $v^+ \approx v^-$ approximation can not be used. Instead the analysis must use the opamp equation (4) at a suitable point. The analysis proceeds as follows:

$$v^- = v_o \frac{R_1}{R_1 + R_2} \text{ as before} \tag{14}$$

$$v^+ = v_i \text{ as before} \tag{15}$$

Now the opamp equation must be used to relate v^+, v^- and v_o.

$$v_o = A_v \left(v^+ - v^-\right) = A_v \left(v_i - v_o \frac{R_1}{R_1 + R_2}\right) \tag{16}$$

transposing for v_i

$$v_o \left[\frac{1}{A_v} + \frac{R_1}{R_1 + R_2}\right] = v_i \tag{17}$$

then the gain is

$$\frac{v_o}{v_i} = \frac{1}{\frac{1}{A_v} + \frac{R_1}{R_1 + R_2}} \tag{18}$$

Note that if $A_v \to \infty$, $1/A_v$ becomes negligible and (18) becomes (9).

Inverting Amplifier

Start as before by summing the current at the inverting (v^-) node.

$$i_i + i_f = 0 \text{ or } \frac{v_i - v^-}{R_1} + \frac{v_o - v^-}{R_2} = 0 \tag{19}$$

which can be transposed to give v^-

$$v^- = v_i \frac{R_2}{R_1 + R_2} + v_o \frac{R_1}{R_1 + R_2} \tag{20}$$

the circuit diagram makes clear that

$$v^+ = 0 \tag{21}$$

Now use the opamp equation (substitute (20) and (21) into (4))

$$v_o = A_v \left(0 - \left[v_i \frac{R_2}{R_1 + R_2} + v_o \frac{R_1}{R_1 + R_2}\right]\right) \tag{22}$$

separating v_o and v_i

$$v_o \left[\frac{1}{A_v} + \frac{R_1}{R_1 + R_2}\right] = -v_i \frac{R_2}{R_1 + R_2} \tag{23}$$

and finally the gain is

$$\frac{v_o}{v_i} = -\frac{\dfrac{R_2}{R_1+R_2}}{\dfrac{1}{A_v}+\dfrac{R_1}{R_1+R_2}} \tag{24}$$

Again if $A_v \to \infty$, v_o/v_i is given by (13).

Circuit Input Resistance

The input to the non-inverting circuit goes directly to the opamp so the circuit input resistance is the same as that of the opamp i.e. very high. The inverting circuit is slightly different. Taking the $A_v \to \infty$ case, and i_i of v_i/R_1 flows from the source. Input resistance is the ration of applied signal current drawn i.e. $v_i/i_i = R_1$. this is typically likely to be a few kΩ which makes inverting amplifiers unsuitable as amplifiers of signal derived from sources with a large Thévenin resistance.

The Unity Gain Buffer

The unity gain buffer is a special case of the non-inverting amplifier. Here $v^- = v_o$ so the opamp equation becomes

$$v_o = A_v\left(v^+ - v^-\right) = A_v\left(v_i - v_o\right) \tag{25}$$

transposing for voltage gain

$$\frac{v_o}{v_i} = \frac{1}{\dfrac{1}{A_v}+1} = \frac{A_v}{1+A_v} \tag{26}$$

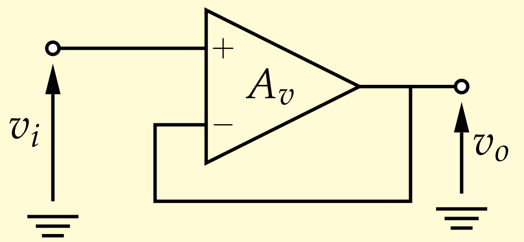


Figure 5: Unity gain buffer.

If A_v is large. v_o/v_i is very close to unity. The circuit is used to isolate high impedance sources from low impedance loads; it has a high power gain.

Circuits with Multiple Inputs

Operational amplifier circuits can have many inputs, limited only by the practicality of connections and space. Multiple input circuits considered in this course are a kind of summing amplifier and a kind of subtractor or difference amplifier. A general method for multiple input circuits will be given. When considering opamp circuits with more than one input in this course we will assume $A_v \to \infty$.

Summing Amplifier

Many analogue audio "mixers" use the circuit shape shown in Fig. 6. It is superior in several respects to a summing amplifier based on a non-inverting topology. The nature of the superiority is left as an exercise for the curious mind - it's to do with impedances. Assume $A_v \to \infty$ so $v^- \to$ virtual earth i.e 0 V. This assumption is always valid in a practically sensible multiple input circuit. Then,

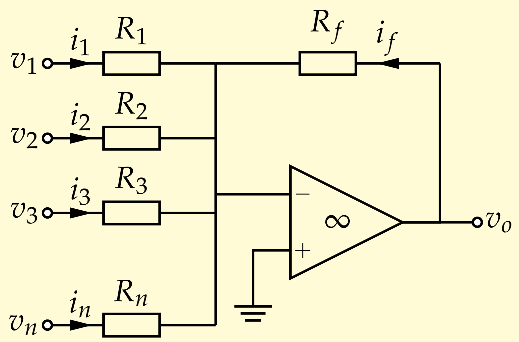


Figure 6: Summing amplifier.

$$i_f + i_1 + i_2 + i_3 + \cdots + i_n = 0 \quad (27)$$

using Ohm's law

$$\frac{v_o}{R_F} + \frac{v_1}{R_1} + \frac{v_2}{R_2} + \frac{v_3}{R_3} + \cdots + \frac{v_n}{R_n} = 0 \quad (28)$$

transposing for v_o

$$v_o = -\left[v_1\frac{R_F}{R_1} + v_2\frac{R_F}{R_2} + v_3\frac{R_F}{R_3} + \cdots + v_n\frac{R_F}{R_n}\right] \quad (29)$$

Subtractors or Difference Amplifiers

The subtractor shown in Fig. 7 can be solved by several methods. Since $A_v \to \infty$, $v^+ = v^-$ so here we will work out v^+ and v^- and then equate them to get v_o in terms of v_1 and v_2. Summing currents at the v^- node

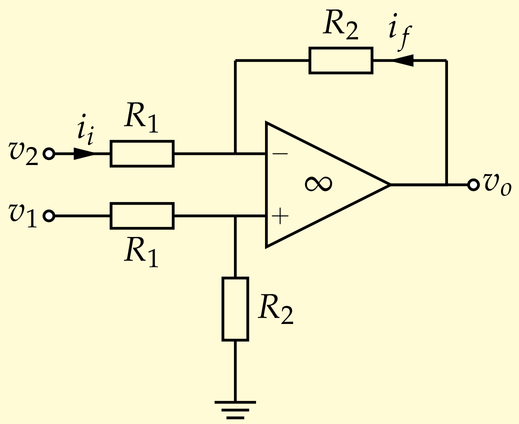


Figure 7: Subtracting amplifier.

$$i_i + i_f = 0 \quad (30)$$

Using Ohm's law

$$\frac{v_2 - v^-}{R_1} + \frac{v_o - v^-}{R_2} = 0 \quad (31)$$

and this can be transposed to give

$$v^- = v_2\frac{R_2}{R_1 + R_2} + v_o\frac{R_1}{R_1 + R_2} \quad (32)$$

v^+ is a potentially divided version of v_1

$$v^+ = v_1 \frac{R_2}{R_1 + R_2} \tag{33}$$

equating v^+ and v^-

$$v_2 \frac{R_2}{R_1 + R_2} + v_o \frac{R_1}{R_1 + R_2} = v_1 \frac{R_2}{R_1 + R_2} \tag{34}$$

transposing for terms in v_o on the left

$$v_o \frac{R_1}{R_1 + R_2} = v_1 \frac{R_2}{R_1 + R_2} - v_2 \frac{R_2}{R_1 + R_2} \tag{35}$$

cancelling

$$v_o = \frac{R_2}{R_1} (v_1 - v_2) \tag{36}$$

Note that the accuracy of the subtraction depends upon the matching of the tow R_1s and R_2s.

General Multiple Input Circuits

The subtractor circuit can be generalised to allow more than two inputs. Such a circuit is shown in Fig. 8. It could be analysed by finding v^+ and v^- and equating them or by using the principle of superposition. Superposition has the advantage that at each stage the circuit is reduced to a familiar single input circuit. Consider first the output due to v_1. In this case v_2, v_3 and v_4 are grounded the circuit becomes that of Fig. 9a

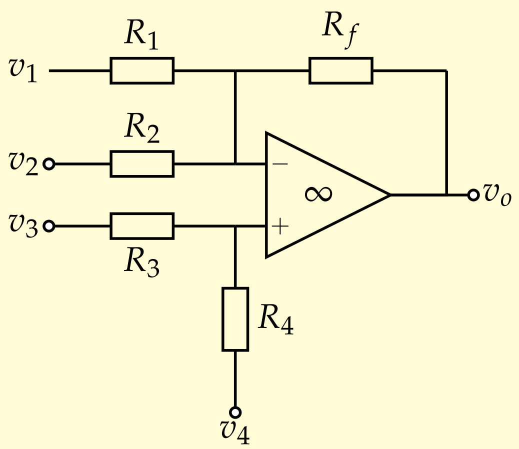


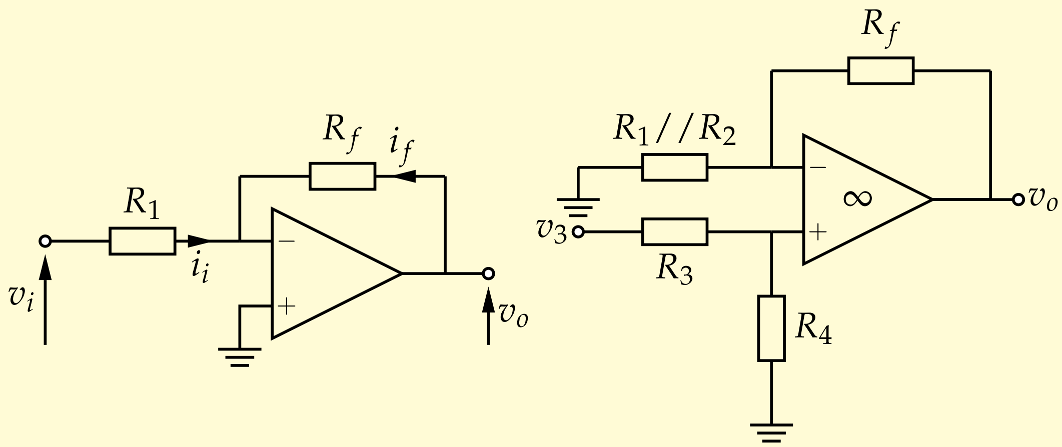
Figure 8: Subtracting amplifier.

Note that since both v_3 and v_4 are zero, v^+ is zero and v^- is a virtual earth. since v^- is a virtual earth, no current flow through R_2 so it has no effect on the circuit.

$$v_o|_{v_1} = v_1 \left(-\frac{R_F}{R_1} \right) \tag{37}$$

By a very similar argument

$$v_o|_{v_2} = v_2 \left(-\frac{R_F}{R_2} \right) \tag{38}$$



(a) Partial circuit for superposition of v_1. (b) Partial circuit for superposition of v_3.

Figure 9: Partial circuits for superposition of the multiple input opamp circuit.

The output due to v_3 leads to a more complicated partial circuit shown in Fig. 9b. Here $v_1 + v_2$ are grounded so R_1 is effectively in parallel with R_2. v^+, the non-inverting amplifier input is a potentially divided version of v_3 so

$$\frac{v_o}{v^+} = \frac{R_F + R_1//R_2}{R_1//R_2} \tag{39}$$

this is just the non-inverting opamp gain. And potential division

$$\frac{v^+}{v_3} = \frac{R_4}{R_3 + R_4} \tag{40}$$

defines the relationship between v^+ and v_3. Bringing the two together

$$\frac{v_o}{v_3} = \frac{v_o}{v^+} \cdot \frac{v^+}{v_3} = \frac{R_4}{R_3 + R_4} \cdot \frac{R_F + R_1//R_2}{R_1//R_2} \tag{41}$$

v_o due to v_3 is then

$$v_o|_{v_3} = v_3 \frac{R_4}{R_3 + R_4} \cdot \frac{R_F + R_1//R_2}{R_1//R_2} \tag{42}$$

By a very similar argument

$$v_o|_{v_4} = v_4 \frac{R_3}{R_3 + R_4} \cdot \frac{R_F + R_1//R_2}{R_1//R_2} \tag{43}$$

The total output voltage is the sum (superposition) of the results so far

$$v_{o(\text{total})} = v_o|_{v_1} + v_o|_{v_2} + v_o|_{v_3} + v_o|_{v_4} \tag{44}$$

Remember that if any of the inputs have d.c. and a.c. parts, those two parts can be treated as separately due also to the superposition principle.